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CALCULATION OF ELLIPTIC ELEMENTS OF THE SYSTEM OF *Y CYGNI.*

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In spite of the very bad weather that prevailed almost continuously in the autumn months, Dr. Bergstrand and I succeeded in observing the following minima of *Y Cygni* in the year 1898:

Epoch	G. M. T.	Observer
	d h m	
2774	1898 April 26 12 5.9	D.
2776	April 29 11 39.4	D.
2854	Aug. 24 8 31.6	D.
2861	Sept. 3 13 30.4	D.
2895	Oct. 24 12 23.6	B.
2902	Nov. 4 5 44.5	D.
2902	Nov. 4 6 5.7	B.
2906	Nov. 10 6 1.7	B.
2907	Nov. 11 11 56.7	B.

On comparing these observations with the ephemeris which I communicated in the *Vierteljahrschrift der Astronomischen Gesellschaft* for 1897, I found the following differences between observation and prediction:

EVEN EPOCHS	
Epoch	O—C
	h m
2774	—0 53.1
2776	—1 14.3
2854	—0 54.8
2902	—1 21.2
2902	—1 42.3
2906	—1 15.7
Mean 2852	—1 13.6

ODD EPOCHS	
Epoch	C—O
	h m
2861	+1 0.2
2895	+1 22.6
2907	+1 26.4
Mean 2887	+1 16.4

From these the following two normal minima were derived:

Epoch $= 2852$; minimum $= 1886.0 + 4616^d.349$; $O—C = —0^d.052$

Epoch $= 2887$; minimum $= 1886.0 + 4668^d.529$; $O—C = +0^d.053$

The considerable deviations of the earlier formulae, which are negative for the even normal minimum and positive for the odd minimum, and are numerically almost precisely equal, show that the observations can no longer be satisfied by formulae which include only the first power of the epoch number. I have therefore derived the following formulae by the method of least squares:

$$\left.\begin{aligned}\text{Even minima} &= 1886.0 + 343^d.4670 + 1^d.498276\,E \\ &\qquad - 0^d.000\,000\,0255\,E^2 \\ \text{Odd minima} &= 1886.0 + 343^d.4131 + 1^d.498076\,E \\ &\qquad + 0^d.000\,000\,0190\,E^2\end{aligned}\right\} \quad (1)$$

The coefficients of E^2 in the two formulae have therefore almost the same numerical value but opposite signs, as was to be expected.

In my earlier essays on this star[1] I have expressed the opinion, now probably generally adopted, that the system of *Y Cygni* consists of two equally large and bright components whose mutual occultations produce the light variation of the stars. At the same time I have, however, also asserted that the line of apsides of the orbits of the two stars turns in the plane of the orbit in such a way that the angle between these two lines has constantly been increasing since the discovery of the variability of the star by Chandler in 1886, when the line of apsides

[1] For instance, *Sur les éléments de l'étoile variable Y Cygni*, Stockholm, 1892.

nearly coincided with the line of sight. The last terms in the equations (1) prove that this angle was in 1886 a straight line and now exceeds 90°, so that the line of apsides now approaches the line of sight. Since the last term should evidently have one and the same coefficient in the two formulae (1), I have taken the mean of them, adopting therefore for these terms the values $-0^d.000\,000\,022\,E^2$ and $+0^d.000\,000\,022\,E^2$. After introducing these values in the equations of condition I again solved and thus obtained :

$$\left.\begin{aligned}\text{Even minima} &= 1886.0 + 343^d.4686 + 1^d.498267E \\ &\qquad - 0.000\,000\,022\,E^2 \\ \text{Odd minima} &= 1886.0 + 343^d.4175 + 1^d.498068\,E \\ &\qquad + 0^d.000\,000\,022\,E^2\end{aligned}\right\} \quad (2)$$

Direct comparison with the normal minima results as follows :

EVEN MINIMA		ODD MINIMA	
Epoch	O—C	Epoch	O—C
12	$-0^d.008$	191	$-0^d.013$
418	+0 .014	705	+0 .014
1184	—0 .021	1187	+0 .007
1640	+0 .019	1311	+0 .007
2208	—0 .006	1913	—0 .010
2858	+0 .003	2371	—0 .012
		2887	+0 .006

The agreement between the formulae and the normal minima is therefore as good as could be asked for. Nevertheless it is clearly possible that this year's minima will no longer agree well, for formulae are not the natural expression of these phenomena and the more terms of higher order that we employ, so much the sooner will terms of a yet higher order be necessary, until finally the formulae are wholly insufficient for furnishing even a rough approximation.

I prefer, therefore, to adopt now the only rational method and to calculate the actual orbital elements. It is obvious at the beginning that a perturbation term, namely the revolution of the line of apsides in the plane of the orbit, must be taken into account in the computation. On the other hand it is equally

obvious that the longitude of the node and the inclination of the orbit plane to the line of sight disappear: the latter must be regarded as zero, and the former has no effect on the light variation of the star and can therefore never be determined.

In order to find a method of determining the orbit I shall show first how the epochs of minima can be computed when the orbital elements are known. It has already been stated that the line of apsides coincided with the line of sight a short time before the first observed minimum. Whether the even or the odd minima then occurred near the perihelion passage, it is quite impossible to decide, and for the present research is not of over much importance. We shall assume, however, that in 1886 the even minima occurred near perihelion, since according to Chandler's observations in 1886 and 1887 there is some probability that the occultations were of longer duration in 1887 than in 1886.

Let Σ be the sidereal period of the star and μ be its mean daily sidereal motion, while U denotes the anomalistic period, and n the mean daily anomalistic motion. Further, let A be the number of tropical years in which the line of apsides turns through 360°. Then $\frac{365.24222}{\Sigma} A$ and $\frac{365.24222}{U} A$ are respectively the number of sidereal and anomalistic revolutions which will occur in A years. The former number is evidently greater by one unit than the latter, and consequently

$$\frac{A}{\Sigma} - 1 = \frac{A}{U}, \quad \text{or} \quad \frac{A - \Sigma}{\Sigma} = \frac{A}{U},$$

whence

$$U = \frac{A}{A - \Sigma} \Sigma . \tag{3}$$

This equation gives us U when Σ and A are known. Further, let 2ω be the angle which the line of apsides turns in the same direction as the stars during one sidereal revolution, t_0 be the time when the line of apsides coincided with the line of sight, e the eccentricity of the orbit, ϕ the angle of eccentricity, and let M, E, v, r, and a have the ordinary significance, as in Gauss'

Theoria Motus, or in Oppolzer's *Lehrbuch zur Bahnbestimmung der Kometen und Planeten*.

At the time t_0, $v = 0$; but at a minimum occurring at the time t, the angle between the line of apsides and the line of sight was equal to $\frac{t - t_0}{\Sigma} \cdot 2\omega$, and the true anomaly at that time was equal to $-\frac{t - t_0}{\Sigma} \cdot 2\omega = -2ma$. But we have

$$dM = \frac{r^2}{a^2 \cos\phi}\, dv = \frac{a^2 \cos^4\phi}{a^2 \cos\phi\,(1 + e\cos v)^2}\, dv = \frac{\cos^3\phi}{(1 + e\cos v)^2}\, dv\ .$$

In order to determine M from this, the equation must be integrated between the limits 0 and $-2m\omega$, or, what is the same thing, between $2m\omega$ and 0. Consequently

$$M = \cos^3\phi \int_{2m\omega}^{0} \frac{dv}{(1 + e\cos v)^2}\ . \qquad (4)$$

But we have

$$\int \frac{dx}{(a + b\cos x)^n} = \frac{A\sin x}{(a + b\cos x)^{n-1}} + \int \frac{B + C\cos x}{(a + b\cos x)^{n-1}}\, dx\ ,$$

where

$$A = \frac{1}{n-1} \cdot \frac{b}{b^2 - a^2}\ ,\quad B = \frac{a}{a^2 - b^2}\ ,\quad C = \frac{n-2}{n-1} \cdot \frac{b}{b^2 - a^2}\ ,$$

whence

$$\int \frac{dv}{(1 + e\cos v)^2} = \frac{e}{e^2 - 1} \cdot \frac{\sin v}{1 + e\cos v} + \frac{1}{1 - e^2} \int \frac{dv}{1 + e\cos v}\ . \qquad (5)$$

We have, further, (for $b < a$),

$$\int \frac{dx}{a + b\cos x} = \frac{2}{a\sin\beta} \cdot \text{arc tan}\left[\tan\frac{1}{2}\beta \tan\frac{1}{2}x\right]\ ,$$

where

$$b = a\cos\beta\ ,\ \text{or}\ \cos\beta = \frac{b}{a}\ ;$$

therefore,

$$\int \frac{dv}{1 + e\cos v} = \frac{2}{\sqrt{1 - e^2}} \cdot \text{arc tan}\left[\sqrt{\frac{1 - e}{1 + e}} \tan\frac{1}{2}v\right]\ , \qquad (6)$$

and by introducing the values from (5) and (6) in (4), there results

$$\cos^3\phi \int \frac{dv}{(1 + e\cos v)^2} = 2\ \text{arc tan}\left[\sqrt{\frac{1 - e}{1 + e}} \tan\frac{1}{2}v\right] - e\sqrt{1 - e^2} \cdot \frac{\sin v}{1 + e\cos v}\ .$$

On introducing the limits we finally obtain

$$M = -2 \operatorname{arc tan}\left[\tan\left(45^\circ - \frac{1}{2}\phi\right)\tan m\omega\right] + \sin\phi\cos\phi\,\frac{\sin 2m\omega}{1 + e\cos 2m\omega}\,. \tag{7}$$

It has been assumed in the above that $v = 0$ at the time t_0. This means that at that time a minimum should also occur. In view of the smallness of ω we may so assume t_0 that this time does coincide with a minimum. Since $\omega < 0^\circ.04$, no error in excess of $0^d.001$ can be thereby introduced in the calculated minima, as will be shown later. This accuracy is consequently more than sufficient for the first computation of the elements.

Now in equation (7) M by no means designates the actual total change of the mean anomaly occurring in the interval $t - t_0$, but on the contrary it is as much less than $\frac{t - t_0}{\Sigma}\,.\,360^\circ = m\,.\,360^\circ$ as the mean anomaly of the star has increased. If M_t is the actual total increase of the mean anomaly, then

$$M_t = m\,.\,360^\circ - 2 \operatorname{arc tan}\left[\tan\left(45^\circ - \tfrac{1}{2}\phi\right)\tan m\omega\right] + \sin\phi\cos\phi\cdot\frac{\sin 2m\omega}{1 + e\cos 2m\omega}\,.$$

But

$$M_t = n\,(t - t_0)\,,$$

whence

$$n\,(t - t_0) = m\,.\,360^\circ - 2 \operatorname{arc tan}\left[\tan\left(45^\circ - \frac{1}{2}\phi\right)\tan m\omega\right] + \sin\phi\cos\phi\,\frac{\sin 2m\omega}{1 + e\cos 2m\omega}\,,$$

or,

$$t = t_0 + mU - \frac{2}{n}\operatorname{arc tan}\left[\tan\left(45^\circ - \frac{1}{2}\phi\right)\tan m\omega\right] + \frac{\sin\phi\cos\phi}{n}\cdot\frac{\sin 2m\omega}{1 + e\cos 2m\omega}\,.$$

Here m is one half the number of the epoch reckoned from t_0. It will be more convenient to introduce in place of this the numbers of the epochs hitherto employed. Therefore let $-E_0$ be the number of the epoch corresponding to t_0, and E be the

epoch for which the computation is to be made. Then we have

$$t = t_0 + (E+E_0)\frac{U}{2} - \frac{2}{n} \text{ arc tan } \left[\tan\left(45^\circ - \frac{1}{2}\phi\right) \tan (E+E_0)\frac{\omega}{2}\right.$$
$$+ \frac{\sin\phi\cos\phi}{n\sin 1^\circ} \cdot \frac{\sin(E+E_0)\omega}{1+e\cos(E+E_0)\omega}\,. \tag{8}$$

This equation holds good for the even minima. We may proceed in a similar way for the odd minima. Let t_1 be the time at which the minimum occurs. Then at the time t_1 the angle between the line of apsides and the line of sight is $\frac{t_1 - t_0}{\Sigma} \cdot 2\omega$, and the true anomaly is

$$v = 180^\circ - \frac{t_1 - t_0}{\Sigma} 2\omega = \pi - 2m_1\omega\,.$$

In order to find M from this we have to integrate the following equation:

$$M = \cos^3\phi \int_0^{\pi - 2m\omega} \frac{dv}{(1+e\cos v)^2}\,. \tag{9}$$

Since, however, as before,

$$\cos^3\phi \int \frac{dv}{(1+e\cos v)^2} = 2 \text{ arc tan} \left[\tan\left(45^\circ - \frac{1}{2}\phi\right) \tan\frac{1}{2}v\right]$$
$$- \sin\phi\cos\phi \frac{\sin v}{(1+e\cos v)^2}\,,$$

we only have to introduce the new limits, and obtain

$$M = 2 \text{ arc tan} \left[\tan\left(45^\circ - \frac{1}{2}\phi\right) \cot m_1\omega\right]$$
$$- \sin\phi\cos\phi \frac{\sin 2m_1\omega}{1 - e\cos 2m_1\omega}\,. \tag{10}$$

The M in this equation now indicates how much more than a certain multiple of 360° the mean anomaly has increased since the passage of perihelion which occurred at the time t_0. Since M is less than 360°, we therefore have to set

$$n(t_1 - t_0) = \frac{E_1 + E_0}{2} \cdot 360^\circ + 2 \text{ arc tan} \left[\tan\left(45^\circ - \frac{1}{2}\phi\right) \cot m_1\omega\right]$$
$$- \sin\phi\cos\phi \frac{\sin 2m_1\omega}{1 - e\cos 2m_1\omega}\,,$$

or, if we introduce the number of the epoch, here $\epsilon_1 = \epsilon + 1$, as before,

$$t_1 = t_0 + (E_1 + E_0 - 1)\frac{U}{2} + \frac{2}{n}\text{arc tan}\left[\tan\left(45^\circ - \frac{1}{2}\phi\right)\cot\left(E_1 + E_0\right)\frac{\omega}{2}\right]$$
$$- \frac{\sin\phi\cos\phi}{n\sin 1^\circ} \cdot \frac{\sin\left(E_1 + E_0\right)\omega}{1 - e\cos\left(E_1 + E_0\right)\omega}\,. \qquad (11)$$

After the formulae for the prediction of the minima from the elements have been developed in this way, we must now show how the elements t_0, ω, n, and e may be computed. I start here from formula (2), which represents the observations very well indeed. We must note here that E cannot denote in both formulae one and the same number. In order that this may be the case for any selected epoch, we may transcribe the first formula (2) in the following way:

$$\text{Even minima} = 1886.0 + 343^d.4686 + 1^d.498267\,(E \mp 1)$$
$$- 0^d.000\,000\,022\,(E^2 \mp 2E + 1)\,,$$

$$\text{or even minima} = 1886.0 + 343^d.4686 \mp 1^d.4983 + [1.498267 \mp$$
$$0.000\,000\,044]\,E - 0.000\,000\,022\,E^2\,.$$

Consequently we have for the two even minima which immediately precede or follow the odd minimum whose epoch $= E$:

$$\text{Even minimum (Epoch} = E - 1) = 1886.0 + 341^d.9703$$
$$+ 1^d.498267\,E - 0^d.000\,000\,022\,E^2\,. \qquad (12)$$

$$\text{Even minimum (Epoch} = E + 1) = 1886.0 + 344^d.9669$$
$$+ 1^d.498267\,E - 0^d.000\,000\,022\,E^2\,. \qquad (13)$$

$$\text{Odd minimum (Epoch} = E) = 1886.0 + 343^d.4175$$
$$+ 1^d.498068\,E + 0^d.000\,000\,022\,E^2\,. \qquad (14)$$

Now let T_1 be the difference between the following even and the preceding odd minimum, or consequently the difference of equations (13) and (14), T_2 on the other hand be the difference between the following and the preceding even minimum, consequently (14) minus (12), and we obtain:

$$\left.\begin{array}{l} T_1 = 1^d.5494 + 0^d.000\,199\,E - 0^d.000\,000\,044\,E^2 \\ T_2 = 1^d.4472 - 0^d.000\,199\,\,E + 0.000\,000\,044\,E^2 \end{array}\right\}. \qquad (15)$$

If we place

$$\theta = T_1 - T_2 \qquad (16)$$

we get

$$\theta = 0^d.1022 + 0^d.000\,398\,E - 0^d.000\,000\,088\,E^2\,. \qquad (17)$$

In order to find when the line of apsides was perpendicular to the line of sight, we must now determine the value of E for which θ is a maximum. Consequently, we differentiate equation (17)

$$\frac{d\theta}{dE} = 0^d.000\,398 - 0^d.000\,000\,176\,E\,.$$

If we now place the differential quotient equal to zero we get

$$E = 2261\,. \tag{18}$$

Introducing this in equation (17), we obtain the maximum value

$$\theta \max = 0^d.5522\,. \tag{19}$$

In order to determine the epoch when the line of apsides coincided with the line of sight, we must place θ equal to zero in equation (17). Thus we obtain

$$E_0 = -244\,, \tag{20}$$

with a residual error of $-0^d.0001$. The line of apsides consequently turns through 90° during 2505 epochs, or, what is the same thing, through 360° during 5010 sidereal revolutions. In order to determine from this the length of the sidereal revolution we can employ formula (2), since in the neighborhood of the time when the line of apsides is perpendicular to the line of sight the intervals between two consecutive even or odd minima are equal among themselves, and equal to the sidereal period. The calculation gives us:

Even minima	Odd minima
E = 2260 Min. = $3385^d.971053$	E = 2261 Min. = $3387^d.244215$
E = 2262 Min. = $3388^d.967388$	E = 2263 Min. = $3390^d.240550$
$\Sigma =$ $2^d.996335$	$\Sigma =$ $2^d.996335$

In this computation terms not containing E in equations (2), and therefore having no influence on the result, were not included. We therefore have

$$\Sigma = 2^d.996335\,. \tag{21}$$

Hence we get

$$\left.\begin{aligned} A &= 15011^d.638 = 41.10 \text{ years} \\ U &= 2^d.996933 \\ 2\omega &= 0^\circ.071856 \end{aligned}\right\} \tag{22}$$

It now remains to determine the eccentricity of the orbit. If no motion of the line of apsides occurred in the interval from

$E=2260$ to $E=2262$, then $\frac{1}{2}(U-\theta)$ and $\frac{1}{2}(U+\theta)$ would represent the times during which the star moved respectively from $v=270^\circ$ to $v=90^\circ$, and from $v=90^\circ$ to $v=270^\circ$.

In fact the true anomalies change by $180^\circ - \frac{U-\theta}{2} \cdot \frac{2\omega}{U}$ and $180^\circ - \frac{U+\theta}{2} \cdot \frac{2\omega}{U}$ respectively from the epochs 2260 to 2261, and from 2261 to 2262.

But
$$dM = \frac{\cos^3 \phi}{(1+e\cos v)^2} dv \,.$$

An approximate knowledge of the eccentricity is therefore necessary. I assume on the basis of preliminary calculations:

$$e = 0.1456\,; \quad \phi = 8^\circ.3518.$$

In view of the smallness of ω as well as of e we may in both cases place $v=90^\circ$ or $v=270^\circ$, and we get

$$dM = \cos^3 \phi\, dv\,,$$

while

$$\left.\begin{aligned} dv &= -\frac{U-\theta}{U}\omega \\ dv_1 &= -\frac{U+\theta}{U}\omega \end{aligned}\right\} \qquad (23)$$

The dM thus found must now be divided by the mean daily anomalistic motion, that is by $\frac{360^\circ}{U} = n$. I find for n the value

$$n = 120^\circ.1228. \qquad (24)$$

From this we obtain

$$\frac{dM}{n} = 0^d.0003 \text{ and } \frac{dM_1}{n} = 0^d.0002,$$

which are to be added to θ; consequently

$$\theta = 0^d.5527. \qquad (25)$$

From this there results

$$\frac{1}{2}(U-\theta) = 1^d.2221\,; \quad \frac{1}{2}(U+\theta) = 1^d.7748, \qquad (26)$$

which are the intervals during which the radius vector sweeps over the two parts of the orbital ellipse bounded by the parameter.

Now the areas are proportional to the times in which they are described. Therefore, if $2s_{\prime\prime}$ and $2s_{\prime}$ are the areas described in the above times,

$$2s_{\prime} : 2s_{\prime\prime} = \frac{1}{2}(U-\theta) : \frac{1}{2}(U+\theta) .$$

Further, if F be the area of the whole ellipse, we shall have the well known relations:

$$\left.\begin{aligned} 2s_{\prime} &= \frac{1}{2}\pi a^2 \cos\phi + \frac{1}{2}\pi a^2 \left[\frac{2\phi + \sin 2\phi}{180^\circ}\right] \cos\phi , \\ 2s_{\prime\prime} &= \frac{1}{2}\pi a^2 \cos\phi - \frac{1}{2}\pi a^2 \left[\frac{2\phi + \sin 2\phi}{180^\circ}\right] \cos\phi , \\ F &= \pi a^2 \cos\phi . \end{aligned}\right\}$$

Hence we have

$$\left.\begin{aligned} \frac{\frac{1}{2}(U+\theta)}{U} &= \frac{2s_{\prime}}{F} = \frac{1}{2} + \frac{1}{2}\left[\frac{2\phi + \sin 2\phi}{180^\circ}\right] , \\ \frac{\frac{1}{2}(U-\theta)}{U} &= \frac{2s_{\prime\prime}}{F} = \frac{1}{2} - \frac{1}{2}\left[\frac{2\phi + \sin 2\phi}{180^\circ}\right] , \end{aligned}\right\}$$

or

$$2\phi + \sin 2\phi = \frac{\theta}{U} . 180^\circ . \tag{27}$$

From this is obtained

$$\begin{aligned} \phi &= 8^\circ\!.3577 \\ e &= 0.14535 \end{aligned} \tag{28}$$

It still remains to calculate the last element, $t_\circ$. As an even minimum occurred at this time, we employ formula (8):

$$t = t_\circ + (E + E_\circ)\frac{U}{2} - \frac{2}{n} \text{ arc tan } \left[\tan\left(45^\circ - \frac{1}{2}\phi\right) \tan (E + E_\circ) . \frac{\omega}{2}\right]$$
$$+ \frac{\sin\phi\cos\phi}{n \sin 1^\circ} . \frac{\sin (E + E_\circ)\omega}{1 + e\cos(E + E_\circ)\omega} .$$

The zero epoch is used for E, whence by (2)

$$t = 1886.0 + 343^d.4686.$$

We thus obtain

$$t_\circ = 1886.0 + 343^d.4686 - 244\frac{U}{2} + \frac{2}{n} \text{arc tan}\left[\tan\left(45^\circ - \frac{1}{2}\phi\right) \tan 244\frac{\omega}{2}\right]$$
$$- \frac{\sin\phi\cos\phi}{n \sin 1^\circ} . \frac{\sin (244\omega)}{1 + e \cos (244\omega)} ,$$

or

$$t_0 = 1886.0 + 343^d.4686 - 122U + \frac{2}{120^\circ.1228} \times$$

$$\text{arc tan}\left[\tan 45^\circ - \frac{1}{2}\phi) \tan (122\omega)\right]$$

$$\frac{\sin\phi\cos\phi}{120^\circ.1228 \sin 1^\circ} \cdot \frac{\sin(244\omega)}{1 + e\cos(244\omega)}. \qquad (29)$$

Therefore

$$t^0 = 1885.0 + 342^d.8968. \qquad (30)$$

I have compared all of the normal minima with the elements thus obtained, and have found the following differences between computation and observation :

Epoch	O—C	Epoch	O—C
12	$-0^d.007$	1640	$+0^d.019$
191	-0.022	1913	-0.014
418	$+0.022$	2208	-0.012
705	$+0.008$	2371	-0.021
1184	-0.016	2852	-0.008
1187	$+0.004$	2887	-0.007
1311	$+0.004$		

The deviations of the computations from the observations cannot be considered too large ; but it cannot be denied that the negative values preponderate. I have therefore taken the mean of them all, and find for the correction to the element t_0

$$dt_0 = -0^d.0038.$$

The final elements therefore become:

ELEMENTS OF Y CYGNI.

Principal epoch, $t_0 = 1885.0 + 342^d.8930$
Motion of line of apsides, $\omega = 0^\circ.035928$
Eccentricity, $e = 0.14535$
Anomalistic period, $U = 2^d.996933$

On comparing these elements with the normal positions, I obtained the following result:

Epoch	Normal minimum	O—C	Epoch	Normal minimum	O—C
12	$361^d.440$	$-0^d.004$	1640	$2800^d.586$	$+0^d.023$
191	629 .536	-0.018	1913	3209 .292	-0.010
418	969 .756	$+0.025$	2208	3651 .529	-0.008
705	1399 .580	$+0.012$	2371	3895 .448	-0.017
1184	2177 .365	-0.012	2852	4616 .349	-0.004
1187	2181 .662	$+0.007$	2887	4668 .529	-0.004
1311	2307 .429	$+0.008$			

It is obvious that a still closer representation of the observation can be obtained by slight variations of the elements. Since, however, the residual errors hardly exceed the probable errors, and since we may expect to obtain observations of the even as well as of the odd minima perhaps as soon as in 1900, I prefer to await these observations before I proceed to improve the elements, which, of course, may be accomplished in a manner analogous to the case of the elements of a planetary orbit.

Of the elements, U and e are evidently already included within very narrow limits. The uncertainty in ω may be considered as somewhat greater in comparison to its less magnitude, and t_0 is somewhat uncertain. Moreover, as has been already remarked, it is not possible to decide whether an even or an odd minimum occurred at the perihelion passage at the time t_0. The former has been assumed above, but on rather uncertain premises.

Finally, a fifth element must be calculated in order to be able to predict the duration of occultation of the star during a given minimum. As such element we may select either the duration of occultation at that minimum when the distance of the stars was equal to the semi-major axis of the orbital ellipse; or the ratio between the diameters of the two stars and the semi-major axis; or, finally, the angle which one star subtends as seen from the other when at their mean distance. Since the knowledge of all three of these quantities is interesting, I shall now show how they may be determined. The observational data upon which this computation may be based are still extremely scant, and were only obtained on searching through my observations to see whether there were any which either began so early or ended so late that the half of the duration of the occultation could be inferred with some certainty. Omitting certain isolated and more unreliable data, I find the following nine observations, all of even minima and all in the year 1892, which appear fairly suitable:

Epoch	Half-duration	Epoch	Half-duration
	h m		h m
1152	2 21	1182	2 55
1162	2 31	1184	2 63
1168	2 33	1194	2 40
1168	2 27	1194	2 50
1180	2 54		

Mean $2^h\,41^m$

The whole duration of the occultation was accordingly

$$T' = 0^d.225.$$

Now, since it is clear that occultation begins at the moment when the stars come into contact as seen from the Earth, and does not cease until the star disks are again tangential, and since the stars are of equal size, the one star must have described an arc of the orbit four times as large as its own radius seen from the other star, if we assume, as is universally done, that one star is stationary. Assuming that the radius is equal, at its mean distance, to π, and letting R be the radius vector at minimum, and taking the semi-major axis as unity, and calling B the arc described by the star, we shall get

$$B = \frac{4\pi}{R},$$

or

$$\pi = \frac{RB}{4}. \tag{31}$$

Since at the beginning of the occultation the moving star touches one of the tangents to the stationary star, parallel to the line of sight, the chord joining the points of the orbit where the center of the moving star is situated at the beginning and end of occultation is equal to the sum of the diameters of the two stars, or, which is the same thing, is equal to four times the radius of one star. If r is now the radius of the star we have

$$\frac{2r}{R} = \sin B$$

or

$$r = \frac{R}{2}\sin B. \tag{32}$$

For two different minima we accordingly have, writing dv in place of B,

$$Rdv = R_1 dv_1.$$

According to Kepler's second law, however,

$$\frac{T}{T_1} = \frac{R^2 dv}{R_1^2 dv_1} = \frac{R}{R_1}.$$

If $R = a = 1$, and T is the duration of occultation,

$$T = \frac{T_1}{R_1}.$$

We may now introduce the factor $\frac{1}{1 + e \cos v}$ in place of R, and we get

$$T = T_1 (1 + e \cos v_1) . \tag{33}$$

We therefore obtain for *Y Cygni*

$$T = 0^d.245 . \tag{34}$$

In this case, however, $v = \omega (E + E_0) = 90° + \phi$. Hence, the corresponding mean anomaly is $M = 81°.67$. If M_1 and M_2 indicate respectively the mean anomalies at the beginning and end of the minimum for which $T = 0^d.245$, we have

$$M_2 - M_1 = \frac{0^d.245}{U} . 360° = 29°.44 .$$

Consequently

$$M_1 = 67°.95 \; ; \quad M_2 = 96°.39 .$$
$$v_1 = 84°.26 \; ; \quad v_2 = 112°.38 .$$

and

$$\left.\begin{aligned} \pi &= \frac{1}{4}(v_2 - v_1) = 7° \\ r &= \sin \pi = \frac{1}{8} \end{aligned}\right\} . \tag{35}$$

It is perfectly evident that this whole investigation of the elements T, π, and r is highly uncertain. For one thing, the quantities T', directly found from the observations, are not very sharply determined. But the principal uncertainty is whether a perihelion or an aphelion passage occurred at the chief epoch t_0; and this is of quite a different significance in the determination of the above elements than for those of others. But the values

found for T, π, and r are nevertheless to be regarded as approxi mations to the truth. The general result of the foregoing investigation can therefore be expressed as follows:

The variable star Y Cygni consists of two stars of equal size and equal brightness, which move aboui their common center of gravity in an elliptical orbit whose major axis is eight times the radius of the stars. The period of an anomalistic revolution is 2.996933 days, and the eccentricity is 0.145. A minimum occurred while the stars were at perihelion (?)[1] *at* 21^h 26^m *G. M. T., on December 8, 1885. The line of apsides of the orbit, which then coincided with the line of sight, completes one revolution in the plane of the orbit tn 41.1 tropical years.*

These investigations may well claim some interest. The observations on which they are based were made with the very simplest apparatus, namely with ordinary telescopes, without photometers, but with the use of Argelander's famous method of estimation in grades. Nevertheless, the observations have proven the existence of a binary system hitherto unknown, whose period, shape and position of orbit, and position of line of apsides are now known with a degree of accuracy in part not inconsiderable. Moreover, we know approximately the ratio between the major axis of the orbit and the diameter of the stars, as well as the angular diameter of the one star as seen from the other. Finally, a force has been brought to light which causes the line of apsides to assume all possible positions in the orbital plane in a space of about forty years.

The line of apsides will again coincide with the line of sight in 1906, and then the even minima will occur at the time of aphelion passage, if we have been correct above. It will then be possible, and possibly even earlier, to decide with certainty from observations with photometers and by estimates whether an aphelion or a perihelion passage occurs at this time. As it is my intention to make observations of the even minima occurring at a very favorable season in this year, for the purpose of observing the duration of occultation of the star, I have predicted in the following ephemeris the times when the minima occur,

[1] The uncertainty is whether it was perihelion or aphelion.

but not the duration of the occultation, in order not to prejudice my observations thereby.

EPHEMERIDES FOR 1900.

Even minima		Odd minima	
Epochs	G. M. T.	Epochs	G. M. T.
	d h m		d h m
3186	Jan. 3 17 31	3185	Jan. 2 0 0
3206	Feb. 2 16 37	3205	Feb. 0 23 8
3226	Mar. 4 15 43	3225	Mar. 2 22 17
3246	Apr. 3 14 49	3245	Apr. 1 21 25
3266	May 3 13 55	3265	May 1 20 34
3286	June 2 13 1	3285	June 0 19 42
3306	July 2 12 7	3305	June 30 18 51
3326	Aug. 1 11 13	3325	July 30 18 0
3346	Sept. 0 10 18	3345	Aug. 29 17 9
3366	Oct. 0 9 24	3365	Sept. 28 16 18
3386	Oct. 30 8 29	3385	Oct. 28 15 27
3406	Nov. 29 7 35	3405	Nov. 27 14 36
3426	Dec. 29 6 40	3425	Dec. 27 13 45

It will be seen that the even minima may be observed in *Europe* under especially favorable circumstances from the beginning of July onward. The odd minima can be observed under equally favorable conditions at the same time in *America.*

UPSALA,
February 2, 1900.

ON THE MAGNITUDES OF 919 FIXED STARS DETERMINED FROM SEQUENCES OBSERVED BY SIR JOHN HERSCHEL DURING THE YEARS 1835 to 1838.

I.

By W. DOBERCK.

THE value of astronomical observations increases with their age, especially in case of determinations of magnitudes of fixed stars. Old records would enable us to discover secular variations in those magnitudes.

Sir John Herschel's observations were made nearly seventy years ago with the naked eye, without any instrumental appliances, according to the method of sequences which, with the exception of the very brightest stars, is capable of furnishing results which are as accurate as the latest photometric determinations. Sir John Herschel not only made these observations, but he also partially reduced them and determined magnitudes of stars included in Table I, using as standards the magnitudes given in the catalogue published by the Astronomical Society in 1827, although he mentions that those latter magnitudes are not homogeneous and, at least in individual cases, frequently considerably in error. Gould mentions that the order of the magnitudes given by Herschel differs at present so widely from that recorded by him, that it is not always easy to decide what magnitude on our scale corresponds to his determination, and Sir John Herschel himself expressed the hope that his sequences might be definitively reduced at some future time, when accurate photometric determinations of the magnitudes of fixed stars were available for reference.

Sir John Herschel states that he chose perfectly clear nights (which for this purpose are quite indispensable), but it is found that the results on some nights, especially on December 29, 1837, are not so satisfactory as those obtained on other nights, which may possibly be due to the existence of faint patches of cirrus haze, which were not remarked at the time. Sir John

Herschel found it impracticable to limit himself to the comparison of stars of nearly the same altitude, nor could he always avoid working on moonlight nights. He states that he did not consider it worth while to bestow particular attention upon insignificant stars, not exceeding the sixth magnitude.

The order of the stars in Table I is as definitively arranged by Sir John Herschel in his corrected normal sequence. The same table shows the magnitudes according to the *Uranometria Argentina*,[1] and the equalized magnitudes, which are the means of the star's own magnitude as stated in the former column and those of the two stars immediately preceding and the two immediately following it in order. Taking now the numerical order of each star from the beginning of the table for an abscissa, and erecting, on paper divided into millimeter squares, an ordinate equal to its equalized magnitude, a series of points was obtained. A curve was now drawn as nearly as possible through (or rather among) those points. This curve was then read off, and the reading set down in the last column, the magnitudes shown in which have been used as standards in reducing the individual sequences.

Each of the forty-six sequences was then treated separately in a similar manner. The numerical order of each star from the beginning of the sequence was taken for an abscissa, and an ordinate equal to its provisional magnitude in Table I was dotted down in case of stars included in that table. No ordinate was entered in case of stars not included in the table, except in case of stars fainter than the stars in Table I. In case of the latter their magnitudes as given in the *Uranometria Argentina* were entered as the corresponding ordinates. Curves were then drawn, keeping as close as possible to all the dots as the condition of a continual increase of the ordinate would allow. In general this was very satisfactory as long as the provisional magnitudes were in question, but when it came to the magnitudes taken from the *Uranometria Argentina* the agreement was less satisfactory. The apparent magnitudes of the stars in every sequence were now

[1] When not found in the *U. A.* the magnitudes were taken from the *Uranometria Nova*, and are shown in parentheses.

read off on the curves, and the results are given in Table II. The stars are here distinguished by their numbers in Table III, which contains the final catalogue. The value of a step, or grade (the difference in magnitude between two adjacent stars), is very different in different sequences or in different parts of the same sequence. For stars of the first two magnitudes it amounted in general to about 0.20 of a magnitude, for stars of about the fourth magnitude it came to about 0.08 magnitude or less, while for stars of the sixth magnitude it varied between 0.02 and 0.40 of a magnitude. The ordinate rises quickly at the beginning of a sequence, a natural consequence of the fact that the brighter stars are scarce, then it gradually rises more slowly and either continues to approach parallelism with the axis of abscissae, or, what is more frequent, it again gradually rises steeply. The sequences 10, 14, 15, 17, 19, and 38 were represented by straight lines. The probable error of a magnitude shown in Table II is in case of the second magnitude ± 0.075, of the third ± 0.087, of the fourth ± 0.121, and of the fifth ± 0.204. Herschel observed stars of the first magnitude on an average 11 times, of the second 9 times, of the third 3 times, of the fourth twice, of the fifth once or twice, and of the sixth once only.

The identification of the stars has given great trouble, although I was ably assisted by Mr. J. I. Plummer and Mr. F. G. Figg, and I have not as yet been able to identify the following five stars: *σ Eridani* (1837.874, 3.87), *ψ Columbae* (1838.018, 6.08), *P. Carinae* (1836.179, 5.76), *Cent.* 154 *Bode* (1837.993, 6.32), and *ι Coronae Aust.* (1835.540, 5.91).

Herschel occasionally included the major planets in his sequences with the following results:

Mars, 1836.867: 1.12. *Saturn*, 1836.415: 0.95, 1837.188: 1.00, 1837.272: 1.28, 1838.149: 1.10. *Jupiter*, 1836.867: ÷0.62, 1837.990: ÷0.20, 1837.993: ÷0.33, 1838.149: ÷0.21, 1838.-283: ÷0.44.

Table III shows the mean magnitudes: column 1 indicates the number and column 2 the name of the star according to Gould's nomenclature. The third and fourth columns show the

coördinates of the stars for 1875. The fifth column shows the mean magnitude according to Sir John Herschel (h), and the sixth column shows how often he observed the star (n). The seventh column shows the magnitude which Behrmann (B) assigned to the star in autumn 1866, reduced to the *U. A.* scale by aid of the table in the *Uranometria Argentina* (page 117). The eighth column contains the *U. A.* magnitude. The ninth column shows the *S. M. P.* magnitude taken from *Annals of the Astronomical Observatory of Harvard College*, Volume XXXIV. The tenth column shows the magnitude observed by Mr. A. Stanley Williams in 1885–6 and published in his catalogue of the magnitudes of 1081 stars (London, 1898). The data in columns 9 and 10 have been reduced to *U. A.* scale by aid of the tables given by Mr. Williams on pages 8 and 10 of the introduction to his catalogue. In case the light of a star as seen by the naked eye is made up of several stars separately noted in the *U. A.* or *S. M. P.* their light has been added together and the resulting magnitude entered. Where this has not been done, the magnitude is that of the brightest star only and is placed in brackets [ ]. Where it was not possible to apply any reduction to *U. A.* the magnitude is placed in parentheses ( ). The eleventh column contains the mean of all the magnitudes, but this is given only in cases where it is most probable that the magnitude has not undergone any variation whatsoever during the past seventy years.

These mean magnitudes might serve as standards in future photometric work in the southern hemisphere. They are referred to the *U. A.* scale, and I am by no means satisfied that that is the best scale that could be selected, but then it is so well known that magnitudes observed on any other scale can be easily referred to it, or the magnitudes here given can conveniently be reduced to any other system. Stars possibly variable have been pointed out. They deserve attention. With reference to secular variation, more or less faintly indicated in case of some stars, it appears that nothing definite will be known till further observations are available.

My thanks are due to Mr. J. I. Plummer, and to Mr. F. G. Figg for the willingness with which they have assisted me in carrying out this investigation. The former gentleman especially has devoted much time to calculations in connection with this work.

TABLE I.

Provisional magnitudes based on Herschel's corrected sequences and the *Uranometria Argentina*.

Name	*U. A.*	Equalized Mag.	Provisional Mag.	Name	*U. A.*	Equalized Mag.	Provisional Mag.
α *Canis Majoris*	0.1	(0.1)	0.07	β *Ceti*	2.3	2.16	2.14
α *Argûs*	0.4	(0.4)	0.41	θ *Centauri*	2.2	2.16	2.15
α *Centauri*	0.7	0.64	0.64	β *Aurigae*	(2.0)	2.16	2.15
α *Bootis*	(1.0)	0.82	0.80	κ *Orionis*	2.3	2.10	2.16
α *Aurigae*	(1.0)	0.94	0.93	α *Ursae Minoris*	(2.0)	2.10	2.16
α *Lyrae*	(1.0)	1.04	1.00	α *Pegasi*	(2.0)	2.16	2.17
β *Orionis*	1.0	1.04	1.05	β *Canis Majoris*	2.2	2.10	2.18
α *Canis Minoris*	1.2	1.08	1.07	δ *Orionis*	2.3	2.10	2.19
α *Eridani*	1.0	1.08	1.09	β *Leonis*	(2.0)	2.12	2.21
α *Orionis*	1.2	1.12	1.12	α *Coronae Bor.*	(2.0)	2.16	2.22
α *Tauri*	(1.0)	1.14	1.14	α *Ophiuchi*	2.1	2.20	2.24
β *Centauri*	1.2	1.22	1.18	γ *Centauri*	2.4	2.30	2.26
α *Crucis*	1.3	1.20	1.22	ι *Carinae*	2.5	2.36	2.28
α *Scorpii*	1.4	1.30	1.27	ζ *Puppis*	2.5	2.40	2.31
α *Aquilae*	1.1	1.34	1.34	β *Pegasi*	(2.3)	2.40	2.34
α *Virginis*	1.5	1.42	1.41	ε *Bootis*	(2.3)	2.36	2.36
α *Piscis Australis*	1.4	1.48	1.48	α *Phoenicis*	2.4	2.32	2.39
β *Geminorum*	(1.7)	1.52	1.55	ε *Scorpii*	2.3	2.38	2.42
β *Crucis*	1.7	1.60	1.61	γ *Ursae Majoris*	(2.3)	2.38	2.44
α *Leonis*	(1.3)	1.72	1.67	α *Lupi*	2.6	2.42	2.46
α *Gruis*	1.9	1.74	1.73	β *Ursae Majoris*	(2.3)	2.54	2.48
γ *Crucis*	2.0	1.80	1.77	ε *Centauri*	2.6	2.56	2.50
ε *Orionis*	1.8	1.94	1.83	η *Canis Majoris*	2.9	2.50	2.52
ε *Ursae Majoris*	(2.0)	1.92	1.87	δ *Scorpii*	2.4	2.58	2.53
λ *Scorpii*	2.0	1.92	1.91	δ *Leonis*	(2.3)	2.56	2.54
ε *Canis Majoris*	1.8	1.92	1.95	ζ *Centauri*	2.7	2.48	2.56
α *Ursae Majoris*	(2.0)	1.92	1.99	γ *Corvi*	2.5	2.48	2.57
ζ *Orionis*	1.8	1.92	2.02	η *Centauri*	2.5	2.54	2.59
β *Carinae*	2.0	1.90	2.05	α *Ceti*	2.4	2.48	2.60
η *Ursae Majoris*	(2.0)	2.10	2.07	β *Corvi*	2.6	2.52	2.61
γ *Orionis*	1.7	2.16	2.09	η *Ophiuchi*	2.4	2.64	2.62
γ *Velorum*	3.0	2.18	2.10	π *Puppis*	2.7	2.68	2.63
α *Pavonis*	2.1	2.18	2.11	γ *Virginis*	3.1	2.66	2.65
Carinae	2.1	2.28	2.12	ζ *Ophiuchi*	2.6	2.72	2.66
β *Tauri*	(2.0)	2.12	2.12	β *Scorpii*	2.5	2.74	2.68
α *Trianguli Aust.*	2.2	2.12	2.12	γ *Pegasi*	(2.7)	2.66	2.70
ε *Sagittarii*	2.2	2.12	2.12	δ *Centauri*	2.8	2.76	2.71
θ *Scorpii*	2.1	2.12	2.12	δ *Ophiuchi*	2.7	2.78	2.72
α *Hydrae*	2.1	2.10	2.13	ζ *Sagittarii*	3.1	2.78	2.73
β *Ursae Minoris*	(2.0)	2.10	2.13	κ *Scorpii*	2.6	2.68	2.74
δ *Canis Majoris*	2.1	2.16	2.14	κ *Velorum*	2.7	2.68	2.75
β *Gruis*	2.2	2.14	2.14	β *Herculis*	(2.3)	2.68	2.76
σ *Sagittarii*	2.4	2.14	2.14	α *Leporis*	2.7	2.76	2.76
α *Arietis*	(2.0)	2.16	2.14	β *Librae*	3.1	2.72	2.77
γ *Leonis*	(2.0)	2.18	2.14	η *Bootis*	(3.0)	2.84	2.78
δ *Velorum*	2.2	2.20	2.14	α *Columbae*	2.5	2.84	2.78
ε *Pegasi*	2.3	2.26	2.14	μ *Velorum*	2.9	2.84	2.79
λ *Velorum*	2.5	2.26	2.14	ε *Virginis*	(2.7)	2.84	2.79
γ *Geminorum*	(2.3)	2.28	2.14	α *Librae*	3.1	2.90	2.80
ζ *Ursae Majoris*	(2.0)	2.26	2.14	α *Canum Venat.*	(3.0)	2.84	2.80

TABLE I — *Continued.*

Name	U. A.	Equalized Mag.	Provisional Mag.	Name	U. A.	Equalized Mag.	Provisional Mag.
β Eridani	2.8	2.84	2.81	q Carinae	3.3	3.40	3.32
α Serpentis	2.6	2.76	2.82	o^{2} Canis Majoris	3.4	3.40	3.34
λ Sagittarii	2.7	2.72	2.83	π Hydrae	3.6	3.46	3.36
β Hydri	2.7	2.76	2.84	ν Puppis	3.5	3.52	3.38
β Lupi	2.8	2.80	2.86	α Pictoris	3.5	3.54	3.40
ι Centauri	3.0	2.86	2.87	η Scorpii	3.6	3.52	3.42
δ Sagittarii	2.8	2.90	2.89	σ Puppis	3.5	3.54	3.43
δ Corvi	3.0	2.90	2.92	ε Gruis	3.5	3.46	3.45
θ Carinae	2.9	2.88	2.95	γ Arae	3.6	3.44	3.46
β Arae	2.8	2.98	2.98	α Doradûs	3.1	3.46	3.47
β Leporis	2.9	3.06	3.00	δ Virginis	3.5	3.38	3.48
ε Corvi	3.3	3.04	3.02	ω Carinae	3.6	3.38	3.49
π Scorpii	3.4	3.06	3.04	ε Leporis	3.1	3.52	3.49
α Toucani	2.8	3.12	3.05	p Carinae	3.6	3.54	3.49
α Muscae	2.9	3.10	3.07	δ Crateris	3.8	3.48	3.50
ρ Puppis	3.2	3.10	3.08	c Puppis	3.6	3.54	3.50
γ Lupi	3.2	3.12	3.09	α Reticuli	3.3	3.54	3.50
σ Scorpii	3.4	3.26	3.10	μ Leporis	3.4	3.46	3.50
α Arae	2.9	3.26	3.10	σ Canis Majoris	3.6	3.54	3.51
μ^{1} Scorpii	3.6	3.28	3.11	γ Phoenicis	3.4	3.56	3.51
υ Scorpii	3.2	3.24	3.11	η Virginis	4.0	3.62	3.51
ι^{1} Scorpii	3.3	3.28	3.12	μ Centauri	3.4	3.60	3.51
τ Scorpii	3.2	3.16	3.12	ν Centauri	3.7	3.60	3.51
π Sagittarii	3.1	3.16	3.12	φ Eridani	3.5	3.52	3.51
γ Aquilae	(3.0)	3.12	3.12	κ Ophiuchi	3.4	3.50	3.51
γ Hydrae	3.2	3.08	3.12	δ Lupi	3.6	3.46	3.52
β Trianguli Aust.	3.1	3.08	3.12	μ Serpentis	3.3	3.42	3.52
β Arietis	(3.0)	3.06	3.12	20 Librae	3.5	3.40	3.54
γ Trianguli Aust.	3.1	3.06	3.12	β Serpentis	(3.3)	3.42	3.55
α Hydri	2.9	3.06	3.12	ε Ophiuchi	3.3	3.50	3.56
τ Puppis	3.2	3.04	3.12	χ Carinae	3.7	3.60	3.58
ζ Hydrae	3.1	3.08	3.12	ε Lupi	3.7	3.68	3.60
ι Orionis	2.9	3.10	3.13	o Velorum	4.0	3.74	3.61
υ Carinae	3.3	3.14	3.14	β Toucani	3.7	3.80	3.63
ν Hydrae	3.0	3.12	3.15	ζ Lupi	3.6	3.80	3.65
δ Crucis	3.4	3.18	3.16	κ Canis Majoris	4.0	3.64	3.67
β Canis Minoris	3.0	3.14	3.16	η Lupi	3.7	3.66	3.68
N Velorum	3.2	3.22	3.17	γ Ceti	3.2	3.68	3.70
π^{3} Orionis	3.1	3.14	3.18	α Horologii	3.8	3.66	3.71
λ Centauri	3.4	3.20	3.19	ξ Hydrae	3.7	3.68	3.72
γ Gruis	3.0	3.24	3.20	δ Columbae	3.9	3.82	3.72
κ Centauri	3.3	3.34	3.20	α Carinae	3.8	3.78	3.73
β Muscae	3.4	3.28	3.21	χ Eridani	3.9	3.78	3.74
ζ Virginis	3.6	3.32	3.22	ϕ^{1} Lupi	3.6	3.76	3.76
α Indi	3.1	3.32	3.22	β Virginis	3.7	3.68	3.77
ζ Canis Majoris	3.2	3.22	3.22	ι Lupi	3.8	3.72	3.78
β Phoenicis	3.3	3.16	3.24	λ Hydrae	3.4	3.70	3.80
β Columbae	2.9	3.24	3.24	κ Lupi	4.1	3.72	3.82
ε Hydrae	3.3	3.12	3.26	β Cancri	3.5	3.76	3.84
α Circini	3.5	3.16	3.26	γ Volantis	3.8	3.96	3.86
θ Eridani	2.6	3.22	3.28	k Puppis	4.0	3.94	3.88
ξ Puppis	3.5	3.22	3.30	l Carinae	4.4	3.96	3.91
γ Hydri	3.2	3.20	3.31	α Puppis	4.0	4.00	3.93

TABLE I — *Continued.*

Name	*U. A.*	Equalized Mag.	Provisional Mag.	Name	*U. A.*	Equalized Mag.	Provisional Mag.
ζ^2 *Scorpii*	3.6	3.94	3.94	κ *Eridani*	4.2	4.32	4.21
γ *Toucani*	4.0	3.86	3.96	δ *Toucani*	4.8	4.26	4.22
ψ *Velorum*	3.7	3.84	3.98	δ *Ceti*	4.0	4.16	4.22
μ *Hydrae*	4.0	3.90	3.99	*P Velorum*	4.1	4.18	4.24
o^1 *Canis Majoris*	3.9	3.96	4.00	δ *Muscae*	3.7	4.02	4.24
ι *Hydrae*	3.9	4.04	4.01	ζ *Volantis*	4.3	4.04	4.26
π *Lupi*	4.3	4.00	4.02	γ *Muscae*	4.0	4.04	4.27
ε *Columbae*	4.1	4.06	4.02	μ *Crucis*	4.1	4.20	4.29
ε *Phoenicis*	3.8	4.10	4.03	ν^2 *Canis Majoris*	4.1	4.14	4.30
ζ *Phoenicis*	4.2	4.04	4.04	η *Phoenicis*	4.5	4.24	4.31
δ *Hydri*	4.1	4.18	4.04	*q Velorum*	4.0	4.26	4.32
η *Columbae*	4.0	4.20	4.05	ρ *Lupi*	4.5	4.40	4.34
μ *Lupi*	4.8	4.14	4.06	α *Chamaeleontis*	4.2	4.38	4.35
β *Pictoris*	3.9	4.16	4.06	ξ^2 *Centauri*	4.8	4.36	4.36
β *Doradûs*	3.9	4.14	4.06	β *Pyxidis*	4.4	4.52	4.37
ε *Hydri*	4.2	3.98	4.06	γ *Apodis*	3.9	4.54	4.38
φ *Velorum*	3.9	3.96	4.06	*f Centauri*	5.3	4.48	4.39
u Carinae	4.0	3.96	4.07	ε *Toucani*	4.3	4.40	4.40
α *Pyxidis*	3.8	3.96	4.08	ε *Volantis*	4.5	4.50	4.40
β *Volantis*	3.9	3.94	4.08	α *Apodis*	4.0	4.34	4.41
l Puppis	4.2	3.96	4.09	γ *Doradûs*	4.4	4.30	4.42
ν *Eridani*	3.8	4.10	4.10	δ *Chamaeleontis*	4.5	4.30	4.42
g Eridani	4.1	4.16	4.10	ζ *Toucani*	4.1	4.44	4.44
β *Hydrae*	4.5	4.12	4.11	δ *Doradûs*	4.5	4.48	4.48
υ^1 *Centauri*	4.2	4.14	4.12	θ *Chamaeleontis*	4.7	4.54	4.55
δ *Phoenicis*	4.0	4.24	4.13	β *Chamaeleontis*	4.6	4.64	4.61
β *Reticuli*	3.9	4.14	4.14	ζ *Doradûs*	4.8	4.66	4.66
ζ *Crucis*	4.6	4.08	4.14	λ^1 *Phoenicis*	4.6	4.74	4.73
ε *Crucis*	4.0	4.10	4.15	ε *Reticuli*	4.6	4.76	4.80
κ *Phoenicis*	3.9	4.26	4.16	ε *Doradûs*	5.1	4.86	4.86
φ *Centauri*	4.1	4.16	4.16	γ *Reticuli*	4.7	4.94	4.93
η *Crucis*	4.7	4.22	4.16	η *Toucani*	5.3	5.14	4.99
δ *Volantis*	4.1	4.28	4.17	κ *Reticuli*	5.0	5.12	5.04
σ *Centauri*	4.3	4.34	4.18	ζ^1 *and* ζ^2 *Reticula*	5.6	5.22	5.10
α *Volantis*	4.2	4.24	4.18	*s Eridani*	5.0	5.20	5.15
I Carinae	4.4	4.26	4.19	ζ *Hydrae*	5.2	(4.90)	5.20
ι *Eridani*	4.2	4.36	4.20	β *Horologii*	5.2	(5.20)	5.25

TABLE II.

Apparent magnitudes of stars observed by Herschel in sequences at Feldhausen from July 16, 1835, to February 25, 1838, and on board H. E. I. C. S. Windsor, at sea, from March 28 to April 15, 1838, inclusive, deduced by aid of the *Uranometria Argentina.*

No.	Ep.	Mag.
1	36.535	2.14
2	37.732	5.82
3	35.620	3.87
	36.905	3.82
	37.812	3.99
4	37.732	5.82
5	36.535	2.78
6	35.620	4.39
	36.905	4.15
	37.812	4.47
7	37.732	5.84
8	37.812	4.53
9	35.620	2.78
	36.179	2.76
	.535	2.86
	.541	2.84
	.905	2.77
	37.730	2.72
	.812	2.94
	.993	2.83
	38.015	3.17
	.018	3.23
	.034	2.96
10	35.620	3.97
	36.905	3.84
	37.812	4.36
11	35.620	2.37
	36.535	2.70
	.905	2.56
	37.730	2.61
	.812	2.46
12	37.732	5.57
13	35.620	4.86
	37.812	4.70

No.	Ep.	Mag.
14	35.620	3.68
	36.541	3.67
	.905	3.64
	37.812	3.61
15	37.732	5.83
16	35.620	4.82
	37.812	5.07
17	37.812	5.33
18	36.535	1.99
	.905	2.42
	37.730	2.33
	.732	2.13
	.812	2.07
	.874	2.35
19	35.620	4.34
	37.812	4.34
20	37.732	4.88
21	37.732	5.74
22	37.732	5.70
23	37.732	5.78
24	37.732	5.90
25	37.732	5.66
26	37.812	5.19
27	37.732	5.88
28	37.732	5.88
29	37.732	4.02
30	37.732	5.79
31	37.732	5.88
32	35.620	3.39

No.	Ep.	Mag.
32	36.905	3.43
	37.812	3.48
33	37.732	5.89
34	37.730	3.39
	.732	3.18
	.874	3.18
35	35.620	3.92
	37.812	3.95
36	37.730	5.15
37	37.812	5.11
38	38.283	2.14
	.286	2.22
39	37.730	3.64
	.732	3.55
40	37.730	5.05
	.732	5.64
41	35.620	3.47
	36.905	3.51
	37.812	3.56
42	37.732	5.22
43	37.732	5.87
44	37.732	5.62
45	35.620	3.81
	36.905	3.72
	37.812	4.31
46	37.732	5.91
47	37.732	5.45
48	35.620	1.16
	36.179	0.82
	.434	1.08
	.535	1.00

TABLE II.—*Continued.*

No.	Ep.	Mag.	No.	Ep.	Mag.	No.	Ep.	Mag.
48	36.601	1.06	65	35.620	2.95	80	37.732	5.81
	.867	1.19		36.541	3.10			
	.905	1.24		.905	2.85	81	37.730	5.30
	37.812	0.87		37.730	3.17		.732	5.84
	.957	1.18		.812	3.01			
	.990	1.16		38.015	3.17	82	37.730	4.86
	.993	0.90		.018	3.18			
	38.004	1.05		.034	3.06	83	37.732	4.71
	.015	1.19						
	.018	1.04	66	37.730	3.45	84	37.730	4.14
	.034	1.13		.874	3.61		.812	4.25
49	37.732	5.92	67	37.732	5.10	85	37.730	4.55
50	37.730	3.58	68	37.732	4.44	86	37.812	4.80
	.732	3.31						
			69	36.905	2.28	87	37.812	4.13
51	37.732	5.41		37.730	2.21			
				.812	1.87	88	37.730	3.52
52	37.730	4.95		.874	1.88		.812	3.70
	.732	4.95					.874	3.25
	.874	4.98	70	37.732	5.03			
						89	35.620	4.25
53	37.730	4.45	71	37.732	5.48		36.905	4.05
							37.812	3.74
54	37.732	5.80	72	35.620	3.55		38.018	4.04
				36.541	3.25			
55	37.812	3.07		.905	3.58	90	37.874	4.44
				37.812	3.66			
56	37.812	4.90		38.018	3.77	91	37.730	4.09
							.874	4.02
57	38.018	5.07	73	37.730	5.35			
						92	37.730	4.51
58	35.620	4.73	74	37.732	5.28		.874	4.38
59	37.730	5.25	75	37.732	5.76	93	38.018	5.72
	.732	5.37						
			76	35.620	4.30	94	35.620	4.64
60	35.620	3.74		36.905	4.12		36.905	4.62
	36.905	3.67		37.812	3.74		37.812	4.83
	37.812	3.78		38.018	3.99		38.018	5.12
61	35.620	5.16	77	37.730	4.61	95	37.730	5.20
	37.812	5.03					.874	4.72
			78	37.874	4.12			
62	37.812	4.93				96	37.730	4.04
			79	35.620	4.02		.874	3.77
63	37.732	5.60		.905	3.77		.874	3.90
				37.730	3.88			
64	37.730	3.76		.812	4.19	97	37.812	5.28
	.732	4.14		38.018	4.45			
						98	36.905	3.28

TABLE II—*Continued.*

No.	Ep.	Mag.
98	37.730	3.00
	.812	3.24
	38.018	3.34
99	37.732	5.90
100	37.730	2.92
	.732	2.58
	.812	2.55
	.874	2.61
101	36.905	5.17
	37.812	5.44
	38.018	5.72
102	37.732	5.77
103	37.730	4.24
	.874	4.07
104	37.812	5.39
105	37.732	5.68
106	37.730	3.93
107	37.730	4.91
	.874	4.78
108	37.730	4.81
109	37.732	5.72
110	37.730	3.99
	.874	3.38
111	38.018	4.56
112	36.905	5.08
	37.812	4.77
	38.018	5.05
113	37.874	3.56
114	37.874	3.66
115	37.874	4.91
116	37.732	5.82
117	37.730	3.82
	.732	3.80

No.	Ep.	Mag.
117	37.874	3.61
118	36.905	5.02
	37.812	4.67
	38.018	5.05
119	37.730	4.20
	.874	4.23
120	37.732	5.85
121	37.874	3.96
122	37.730	5.10
	.732	5.52
123	38.018	4.63
124	37.730	3.24
	.874	3.02
125	37.730	3.70
	.732	3.68
	.874	3.31
	.874	3.13
126	37.874	4.42
127	37.732	5.84
128	37.874	5.09
129	35.620	4.77
	37.730	4.76
	38.018	4.60
130	37.874	4.79
131	37.874	4.79
132	37.874	3.13
133	37.732	5.16
	.874	4.42
134	37.730	4.40
	.874	4.18
135	37.874	3.90
136	37.874	5.34

No.	Ep.	Mag.
137	37.730	5.00
138	36.179	3.76
	.905	3.97
	37.812	4.09
	38.004	3.96
	.018	4.17
139	37.730	4.34
	38.018	4.29
140	37.730	4.30
	38.018	4.12
141	37.730	4.66
	.874	4.84
142	35.620	3.62
	36.179	3.38
	.535	3.31
	.541	3.30
	.905	3.01
	37.812	3.28
	38.004	3.45
	.015	3.32
	.018	3.40
	.034	3.23
143	37.732	5.76
144	37.732	5.76
145	37.730	4.71
	.874	4.54
146	38.018	6.08
147	37.874	3.82
148	37.812	4.96
	38.018	4.95
149	37.732	5.71
150	37.812	4.63
	38.018	4.70
151	37.812	5.23
	38.018	5.11
152	38.018	5.69

TABLE II — *Continued.*

No.	Ep.	Mag.
153	38.018	4.90
154	37.874	4.33
155	38.018	3.81
156	36.905	4.45
	37.812	4.39
	38.018	4.48
157	36.179	3.58
	.535	3.52
	.905	3.67
	37.812	3.43
	38.004	3.62
	.015	3.57
	.018	3.62
	.034	3.54
158	36.905	4.81
	37.812	5.14
	38.018	4.92
159	36.869	5.24
160	38.018	4.37
161	38.018	5.69
162	38.018	5.18
163	36.867	1.33
	37.812	1.54
	.957	1.50
	.993	1.16
	38.015	1.38
	.034	1.31
	.237	1.24
	.283	1.47
164	36.869	3.69
	38.018	4.08
165	38.018	4.33
166	36.179	3.41
	.535	3.40
	.905	3.40
	37.812	3.33
	38.004	3.41
	.015	3.52
	.018	3.52

No.	Ep.	Mag.
166	38.026	3.42
	.034	3.47
167	36.869	5.38
	38.018	5.69
168	37.874	4.60
	38.018	4.21
169	38.018	5.69
170	38.018	5.69
171	38.018	4.82
172	38.018	4.77
173	38.018	5.69
174	38.018	5.69
175	36.869	3.99
	38.018	4.25
176	38.015	5.73
177	38.018	6.08
178	38.018	5.69
179	36.869	3.11
180	36.869	4.68
181	37.993	5.24
182	36.869	3.52
183	38.018	5.69
184	38.018	4.85
185	38.018	5.69
186	36.869	3.60
187	36.869	5.00
188	37.993	5.42
	38.015	5.24
189	38.018	5.19

No.	Ep.	Mag.
190	36.869	4.61
	38.018	5.03
191	38.018	5.72
192	38.015	5.99
193	38.018	4.80
194	38.018	6.08
195	36.869	3.48
	37.957	3.50
	38.018	3.57
196	36.867	2.85
	37.812	2.81
	.957	3.22
	38.004	2.75
	.015	3.12
197	38.015	5.99
198	36.869	4.52
	38.018	4.97
199	36.905	4.75
	38.015	4.90
200	37.993	5.56
201	38.015	5.76
202	36.869	4.89
	38.018	4.52
203	36.869	3.56
	37.957	3.62
	38.018	3.62
204	36.869	4.89
	38.018	4.52
205	38.283	0.87
206	36.223	0.99
	.867	1.05
	.869	1.33
	.905	0.96
	37.812	1.17
	.957	0.97
	.990	1.05

TABLE II—*Continued.*

No.	Ep.	Mag.
206	37.993	0.81
	38.004	0.89
	.015	0.96
	.034	0.92
	.149	0.95
	.152	0.94
	.237	0.92
	.283	1.15
207	38.018	5.72
208	36.869	4.10
209	36.869	4.89
	38.018	4.66
210	37.993	5.03
211	36.869	5.55
212	38.018	5.72
213	38.015	5.73
214	38.018	5.72
215	36.867	3.41
	.869	3.56
216	38.034	2.01
	.283	2.17
217	36.223	2.06
	.867	1.68
	.869	1.59
	.905	2.23
	37.957	2.16
	.990	2.15
	.993	1.86
	38.004	2.01
	.015	1.98
	.026	2.08
	.152	1.86
218	38.018	6.08
219	38.015	5.73
220	36.867	2.99
	.869	3.05
	.905	3.25
	37.957	2.93

No.	Ep.	Mag.
220	38.004	2.70
	.015	3.02
	.018	2.78
	.026	3.07
	.034	2.92
221	38.015	5.45
222	36.223	2.15
	.867	2.37
	.869	2.49
	.905	2.60
	37.957	2.42
	38.152	2.34
223	38.018	3.95
224	36.867	2.57
	.869	2.70
	.905	2.68
	37.812	2.64
	.957	2.70
	38.004	2.54
	.015	2.77
	.018	2.63
	.026	2.84
	.034	2.73
225	36.869	3.74
226	36.869	4.35
227	36.867	3.13
	.869	3.00
	37.957	3.04
	38.034	3.01
228	36.223	2.02
	.867	1.74
	.869	1.90
	.905	1.84
	37.957	1.64
	.990	1.63
	.993	1.81
	38.004	1.94
	.015	1.80
	.026	1.84
	.034	1.69
	.152	1.86
229	38.018	5.72

No.	Ep.	Mag.
230	36.535	3.81
	.905	4.22
	37.812	3.86
	.993	3.34
	38.018	3.72
	.034	4.07
231	36.869	4.44
232	36.223	2.09
	.867	1.95
	.869	1.99
	.905	1.96
	37.957	1.95
	.990	1.98
	.993	1.81
	38.004	2.01
	.015	1.86
	.026	1.96
	.034	1.83
	.152	1.98
233	36.867	2.78
	9.05	2.89
	37.812	2.87
	.957	2.82
	38.004	2.80
	.018	2.71
	.026	2.96
	.034	2.82
	.149	2.79
234	38.018	5.72
235	37.993	5.66
236	36.869	3.88
	37.957	3.56
237	38.018	5.09
238	36.869	3.65
	37.957	3.66
239	36.223	2.20
	.867	2.44
	.869	2.22
	.905	2.47
	37.812	2.36
	.957	2.26
	.990	2.18
	38.004	2.42
	.015	2.42

TABLE II — *Continued.*

No.	Ep.	Mag.
240	38.015	3.96
241	36.179	4.33
	.905	4.50
	37.812	4.50
	.993	4.52
242	36.869	3.94
	37.957	3.82
243	36.869	3.16
	.905	3.36
	37.957	3.16
	38.018	3.06
	.026	3.18
244	37.993	5.06
	38.015	4.86
245	38.015	5.73
246	36.223	1.40
	.867	1.05
	.869	1.33
	.905	1.12
	37.812	0.70
	.812	1.02
	.957	1.35
	.993	1.07
	38.004	1.13
	.015	1.28
	.034	1.22
	.152	1.20
	.237	1.14
	.283	1.27
247	38.018	4.87
248	36.905	4.87
	37.812	4.56
	.933	5.18
249	38.283	2.22
250	36.869	3.83
	37.957	3.77
251	38.018	4.99
252	38.018	5.72
253	38.015	5.73

No.	Ep.	Mag.
254	37.272	4.48
	38.018	4.73
255	36.179	5.07
	37.993	5.28
256	37.272	3.89
	38.018	4.42
257	38.018	5.72
258	38.015	5.42
259	37.272	4.76
	38.018	5.01
260	37.993	5.14
261	37.993	5.80
262	37.993	5.00
263	37.993	5.83
264	37.993	5.75
265	37.272	4.48
266	37.993	5.62
267	36.867	3.20
	.869	3.21
	.905	3.09
	37.272	2.94
	38.018	2.92
	.034	3.17
268	36.223	2.18
	.867	2.16
	.869	2.29
	37.957	2.32
	.990	2.22
	38.004	2.48
	.015	2.30
	.026	2.30
	.034	2.31
	.149	2.30
269	37.272	3.67
270	37.993	5.85

No.	Ep.	Mag.
271	36.114	0.43
	.116	0.60
	.179	0.44
	.415	0.41
	.535	0.50
	.905	0.50
	37.188	0.44
	.812	0.45
	.957	0.44
	990	0.40
	.993	0.38
	38.015	0.40
	.056	0.44
	.149	0.41
	.152	0.41
	.237	0.39
	.283	0.48
272	38.015	5.99
273	37.993	6.03
	.993	6.39
274	37.272	5.00
275	37.993	5.69
276	37.993	6.36
277	37.272	6.19
278	37.993	5.69
279	38.015	5.73
280	37.993	5.98
281	37.993	6.01
	38.015	5.99
282	36.223	2.16
	.867	2.23
	38.015	2.25
	.034	2.27
	.152	2.34
	.286	2.14
283	38.034	5.31
284	38.034	4.26
285	36.179	5.07
	38.015	4.82

TABLE II — *Continued.*

No.	Ep.	Mag.	No.	Ep.	Mag.	No.	Ep.	Mag.
286	38.034	4.75	296	38.026	3.42	308	37.993	5.72
				.034	3.41			
287	36.869	3.45		.152	3.38	309	38.015	5.73
	.905	3.17						
	37.270	2.99	297	36.179	5.52	310	36.179	5.07
	.272	3.36		38.015	4.60		38.015	5.49
	38.015	3.37						
	.152	3.34	298	38.149	4.29	311	38.015	5.73
288	38.015	5.73	299	37.272	3.76	312	36.179	5.52
				38.034	3.99			
289	36.116	0.24				313	36.223	2.16
	.179	0.05	300	38.015	5.73		.867	2.09
	.415	0.10					.869	2.07
	.905	0.10	301	36.223	1.88		.905	2.12
	37.957	0.06		.867	1.60		37.272	2.35
	.990	0.10		.869	1.70		.957	2.10
	.993	0.08		.905	1.77		.990	2.10
	38.015	0.07		37.272	1.88		.993	2.14
	.056	0.07		.957	1.76		38.004	2.29
	.149	0.10		.990	1.84		.015	2.09
	.152	0.06		.993	1.81		.018	1.90
	.237	0.16		38.004	2.01		.026	2.12
	.283	0.17		.015	1.74		.034	2.08
				.018	1.72		.056	2.16
290	37.272	5.86		.026	1.73		.149	2.05
				.034	1.74		.152	2.18
291	37.272	5.90		.149	1.80		.237	2.14
				.152	1.95			
292	37.272	3.62		.237	1.86	314	36.179	4.90
	38.034	3.83					38.015	5.73
			302	37.272	6.14			
293	37.272	5.99				315	36.179	4.90
			303	37.272	3.45		38.015	4.96
294	37.272	6.17		38.034	3.75			
						316	36.179	5.07
295	36.179	3.17	304	36.867	3.27		38.015	5.47
	.905	2.93		.869	3.35			
	37.270	3.19		37.272	3.31	317	37.272	4.55
	.272	3.19		38.026	3.42			
	.957	3.28		.034	3.41	318	36.179	4.90
	38.004	2.97		.149	3.24		.869	4.05
	.015	3.02					38.015	4.93
	.152	3.09	305	36.179	5.52			
				38.015	5.73	319	36.179	3.97
296	36.179	3.47					.905	3.75
	.535	3.03	306	36.867	3.61		37.812	3.83
	.905	3.05		38.034	4.60		.993	3.89
	37.812	3.33		.149	4.39		38.004	3.83
	38.004	3.70					.015	4.01
	.015	3.26	307	38.015	5.99		.018	3.91
	.018	3.52					.026	3.68

TABLE II.—*Continued.*

No.	Ep.	Mag.
319	38.034	3.83
	.149	3.86
	.152	3.81
320	37.993	6.41
321	38.115	5.50
322	36.179	5.52
323	36.869	2.82
	37.270	2.75
	.272	2.63
	37.957	2.64
	38.015	2.62
	.026	2.76
	.149	2.69
	.152	2.58
324	36.179	4.33
	.905	4.02
	37.812	4.06
	.993	4.25
	38.152	4.25
325	38.015	5.73
326	36.867	2.65
	.869	2.63
	37.957	2.53
	38.015	2.52
	.026	2.53
	.034	2.54
	.149	2.56
	.152	2.48
327	36.869	2.88
	38.004	3.23
	.015	3.07
	.018	2.98
328	38.015	5.73
329	36.179	5.52
330	36.869	3.42
	37.270	3.43
	.272	3.45
	.957	3.34
	38.015	3.47
	.018	3.46
	.152	3.43

No.	Ep.	Mag.
331	36.223	1.98
	.867	1.82
	38.004	1.70
	.034	2.16
	.286	1.93
332	36.179	5.52
333	37.272	4.69
334	36.179	5.52
	38.015	5.73
335	37.272	4.92
336	36.223	1.17
	.867	1.12
	37.990	1.27
	.993	0.99
	38.004	0.97
	.015	1.08
	.018	0.94
	.034	1.04
	.152	1.08
	.237	1.02
	.283	1.01
337	37.272	5.30
338	36.179	5.76
	38.015	5.99
339	36.869	4.17
	37.272	3.71
	38.149	3.91
340	36.179	5.76
	38.015	5.99
341	38.149	4.88
342	37.272	6.22
343	36.223	1.69
	.867	1.54
	37.993	1.51
	38.004	1.30
	.015	1.54
	.034	1.52
	.149	1.49
	.283	1.56

No.	Ep.	Mag.
344	37.272	5.37
345	37.272	3.99
	38.149	4.12
346	37.272	6.02
347	37.272	3.57
348	36.179	5.52
	37.272	4.84
349	38.015	5.99
350	36.179	4.25
	.905	4.31
	37.812	4.28
	.993	4.10
	38.015	4.21
	.034	4.16
351	36.869	3.30
	37.272	3.41
	38.149	3.18
352	36.179	5.52
353	36.179	4.90
	38.015	4.64
354	37.272	3.80
355	38.015	5.99
356	36.179	4.90
	38.015	5.00
357	36.179	4.90
	38.015	4.77
358	36.179	3.66
	38.015	3.66
	.026	3.60
	.034	3.75
	.149	3.64
	.152	3.57
359	36.179	4.90
	38.015	5.03
360	37.993	6.48

TABLE II — *Continued.*

No.	Ep.	Mag.
361	37.993	6.06
362	36.179	5.07
363	36.116	2.46
	.179	2.40
	.867	2.51
	.869	2.35
	37.270	2.54
	.272	2.54
	.957	2.37
	.990	2.22
	38.015	2.47
	.018	2.48
	.026	2.38
	.034	2.38
	.149	2.45
	.152	2.42
	.237	2.34
364	36.867	2.92
	37.272	3.07
	38.149	2.96
	.152	2.82
365	38.015	5.73
366	38.149	5.16
367	38.015	5.73
368	36.116	1.78
	.179	1.88
	.223	2.11
	.415	1.95
	.867	2.02
	.905	2.12
	37.188	1.76
	.270	1.98
	.270	1.85
	.272	2.06
	.957	2.03
	.990	2.05
	.993	1.92
	38.004	2.08
	.015	1.98
	.026	1.88
	.034	1.93
	.056	1.80
	.149	1.91
	.149	1.97
	.152	2.02

No.	Ep.	Mag.
368	38.237	2.11
369	36.179	5.07
	38.015	5.17
370	37.993	5.10
	38.015	5.20
371	36.179	5.76
372	36.179	4.33
	.905	4.40
	37.812	4.42
	.993	4.57
	38.015	4.30
373	36.179	5.07
	38.015	5.17
374	38.004	3.79
375	36.179	5.76
	37.993	5.10
	38.015	5.20
376	36.179	5.76
	38.015	5.73
377	38.149	4.07
378	36.116	1.90
	.179	1.96
	.223	2.13
	.415	2.02
	.905	2.23
	37.990	2.13
	.993	2.08
	38.015	2.04
	.026	2.04
	.034	1.88
	.056	1.99
	.149	2.00
	.149	2.14
	.152	2.08
	.237	2.08
	.237	2.16
379	36.179	4.90
	37.812	5.51
	.993	4.96
380	36.230	4.32

No.	Ep.	Mag.
380	36.905	4.35
	37.812	4.44
	38.026	4.45
	.152	3.95
381	38.015	5.73
382	37.812	5.58
383	38.015	5.99
384	36.230	4.51
	.905	4.56
	38.026	4.78
	.152	4.61
385	36.179	4.25
	.240	4.10
	.905	3.95
	37.812	4.22
	38.015	4.11
386	36.179	5.52
	38.015	5.99
387	36.179	5.76
	38.015	5.99
388	38.004	4.40
389	36.179	5.76
390	38.004	4.83
391	38.015	5.36
392	36.179	5.52
	38.015	5.38
393	36.179	4.90
	38.015	5.14
394	37.272	4.35
	38.015	4.26
	.034	4.35
	.149	4.77
	.152	4.51
395	36.179	4.33
	38.015	4.52
396	36.179	4.25
	38.015	3.72

TABLE II—*Continued.*

No.	Ep.	Mag.
397	38.004	4.47
398	36.179	5.76
	38.015	5.07
399	38.015	5.99
400	36.179	4.90
401	36.179	5.76
	38.015	5.73
402	37.272	4.03
	38.015	3.86
	.026	4.02
	.149	4.02
	.152	4.10
403	36.179	5.52
404	38.015	5.27
405	36.179	5.76
	38.015	5.52
406	36.179	4.90
	38.015	5.10
407	38.004	3.37
408	38.149	4.94
409	36.116	2.12
	.179	2.28
	.223	2.22
	.415	2.15
	.905	2.33
	37.993	2.20
	38.004	2.22
	.015	2.14
	.018	2.15
	.026	2.19
	.034	2.20
	.149	2.14
	.149	2.18
	.152	2.24
	.237	2.16
410	36.179	4.33
	38.015	4.48
411	36.179	5.07
411	38.015	5.30
412	38.015	5.99
413	37.272	4.35
	38.149	4.45
	.152	4.83
414	36.179	5.52
	38.015	5.73
415	36.179	5.52
	38.015	5.73
416	38.004	3.02
	.018	3.12
	.152	2.89
	.283	3.17
	.286	3.02
417	37.270	5.21
418	38.015	5.73
419	38.149	5.05
420	36.179	4.33
421	36.179	5.52
	38.015	5.40
422	36.179	5.76
	38.015	5.53
423	38.015	5.73
424	38.015	5.73
425	36.179	5.76
	38.015	5.54
426	38.015	5.99
427	36.179	5.76
	38.015	5.99
428	36.179	4.33
	38.015	4.69
429	36.179	4.25
	.240	4.19
	.905	4.19
429	37.812	4.02
	38.015	4.34
430	36.179	2.22
	.415	2.21
	37.270	2.30
	.993	2.55
	38.015	2.20
	.018	2.32
	.026	2.26
	.149	2.20
	.149	2.25
	.152	2.29
431	38.026	5.33
432	37.812	5.00
	38.026	5.33
433	36.179	3.89
	38.015	3.81
434	36.179	4.33
435	36.116	1.78
	.179	1.81
	.223	2.04
	.415	1.88
	.905	2.07
	37.188	1.76
	.270	2.05
	.270	2.08
	.272	2.21
	.957	1.86
	.990	2.01
	.993	1.97
	38.004	2.01
	.015	1.92
	.026	2.00
	.034	1.83
	.056	1.90
	.149	1.88
	.152	2.05
	.237	2.04
436	36.179	4.90
	38.015	5.30
437	38.283	3.03
438	36.116	2.34
	.179	2.34

TABLE II—*Continued.*

No.	Ep.	Mag.
438	36.413	2.39
	.415	2.32
	38.015	2.72
	.026	2.65
	.034	2.64
	.149	2.33
	.152	2.36
	.237	2.34
439	38.026	5.81
440	36.179	5.52
441	36.116	2.76
	.179	2.72
	.415	2.72
	38.015	2.57
	.026	2.46
	.034	2.46
	.149	2.72
	.152	2.78
442	36.223	2.22
	.867	1.88
	37.993	2.34
	38.004	1.78
	.015	2.09
	.018	2.07
	.026	2.22
	.034	2.20
	.056	2.12
	.149	2.00
	.149	2.10
	.152	2.15
	.237	2.08
	.237	2.14
	.283	2.03
	.286	2.10
443	38 004	5.07
444	38.004	4.72
445	37.272	4.13
	38.018	3.86
	.026	3.76
	.149	3.97
	.152	3.68
446	38.026	5.81
447	36.179	3.31

No.	Ep.	Mag.
447	36.415	3.34
	38.026	3.14
	.149	3.10
448	38.026	5.58
449	36.179	5.07
450	37.272	5.44
451	36.179	4.33
	38.026	5.33
452	37.272	5.07
453	37.272	6.09
454	38.004	4.18
	.015	3.91
	.149	4.02
455	38.018	5.16
	.149	5.10
456	36.179	5.07
457	38.018	5.16
	.149	5.10
458	38.026	5.81
459	38.283	3.07
460	37.272	5.78
461	36.179	3.68
	38.026	3.98
462	36.179	3.28
	.415	3.20
	38.015	2.87
	.026	3.18
	.034	3.11
	.149	3.02
	.149	3.14
	.152	3.18
463	37.272	5.56
464	38.004	4.23
	.149	4.23

No.	Ep.	Mag.
465	37.272	5.96
466	37.272	5.14
467	37.272	6.24
468	36.179	3.74
	37.272	4.08
	.993	4.88
469	37.993	6.43
470	38.149	5.21
471	37.993	6.46
472	38.004	4.95
473	37.272	6.07
474	38.149	4.50
475	36.179	1.55
	.223	1.63
	37.993	1.70
	38.026	1.80
	.056	1.67
	.152	1.64
	.237	1.67
	.283	1.64
476	38.026	5.58
477	37.272	5.62
478	38.026	5.33
479	37.272	5.74
480	37.272	5.82
481	37.272	4.29
	38.015	4.56
	.149	4.18
482	38.026	5.81
483	37.272	6.05
484	36.179	3.44
	.415	3.38
	38.015	2.92
	.026	3.52

TABLE II—*Continued.*

No.	Ep.	Mag.	No.	Ep.	Mag.	No.	Ep.	Mag.
484	38.034	3.61	500	36.179	5.07	508	38.149	2.80
	.149	3.41					.152	2.82
	.149	3.44	501	37.272	4.24		.237	2.62
485	36.179	3.55	502	36.179	5.52	509	36.179	5.52
	.415	3.60						
	38.026	3.34	503	36.179	4.33	510	37.188	3.01
	.034	3.29		.240	4.19		38.004	3.28
	.149	3.30		38.026	4.30		.015	3.42
	.149	3.32		.152	4.96		.149	3.02
							.152	3.26
486	36.114	2.12	504	36.179	4.90		.283	3.12
	.223	1.96						
	38.026	2.30	505	36.179	3.05	511	36.179	4.33
	.056	2.06		.415	3 03		.230	4.46
	.152	2.21		38.026	2.92		38.026	4.33
	.237	2.14		.149	2.75		.152	4.03
	.283	2.08		.152	2.97			
	.286	2.07		.237	2.87	512	38.004	3.75
							.149	3.80
487	38.283	2.99	506	36.179	5.76		.152	3.68
488	36.179	5.52	507	36.114	1.75	513	36.179	4.25
				.116	1.50		38.026	3.88
489	36.179	5.07		.179	1.55			
				.223	1.48	514	38.004	4.28
490	36.179	5.76		.415	1.55		.286	4.13
	38.026	5.58		.867	1.47			
				.905	1.54	515	38.283	2.44
491	38.004	4.09		37.188	1.48		.286	2.39
	.149	3.97		.272	1.65			
				.957	0.73	516	38.237	1.93
492	38.149	4.66		.990	0.65		.283	1.86
				.993	0.71		.286	1.95
493	36.179	4.25		38.015	0.81			
	.230	4.28		.018	0.72	517	38.026	5.33
	38.026	4.18		.034	0.80			
				.056	0.97	518	38.026	5.58
494	36.179	5.52		.149	0.83			
				.149	0.94	519	38.026	5.81
495	36.179	4.33		.152	0.80			
				.237	0.82	520	38.283	2.97
496	36.179	5.07		.283	1.38		.286	2.91
	38.026	5.58						
			508	36.179	2.81	521	38.026	4.52
497	36.179	3.52		.415	2.45			
	.415	3.56		37.272	2.80	522	38.026	5 81
	38.026	3.95		38.004	2.88			
				.015	2.77	523	37.270	4.61
498	37.272	5.50		.026	2.76		38.004	4.54
				.149	2.79		.149	4.56
499	38.026	5.81		.149	2.91			

TABLE II—*Continued.*

No.	Ep.	Mag.
524	36.179	5.07
525	36.223	2.54
	38.283	2.60
	.286	2.48
526	38.026	5.81
527	38.286	3.39
528	38.149	4.61
529	38.283	3.45
530	38.004	3.49
	.149	3.47
	.149	3.50
531	36.179	4.33
532	38.286	3.70
533	38.004	4.14
534	36.179	5.52
	38.026	5.33
535	38.026	5.58
536	37.270	3.71
	38.149	3.58
537	36.179	4.90
538	38.026	5.58
539	38.026	5.58
540	36.179	3.49
	.415	3.29
	38.026	3.45
	.149	3.18
541	38.149	5.00
542	38.283	4.30
543	38.026	5.58
544	38.026	5.81
545	38.026	5.58

No.	Ep.	Mag.
546	37.270	5.08
547	36.179	3.94
	.415	3.90
	38.026	3.83
548	36.179	4.90
	38.026	4.78
549	36.179	5.76
	38.026	5.81
550	36.223	1.92
	38.026	2.38
	.056	2.23
	.237	2.16
	.237	2.23
	.283	2.26
	.286	2.30
551	36.179	5.07
	38.026	4.78
552	37.270	4.88
553	38.026	5.81
554	38.149	3.74
	.283	3.40
	.286	3.59
555	36.179	5.52
	38.026	5 58
556	38.004	3.92
557	38.237	2.34
	.283	2.30
	.286	2.43
558	36.179	5.07
	.230	4.66
	38.026	4.37
	.152	5.23
559	36.179	5.07
	38.026	5.33
560	36.179	5.52
	38.026	5.58
561	36.179	5.76

No.	Ep.	Mag.
562	36.179	4.90
	.240	4.16
	38.026	4.78
563	36.116	2.76
	.179	2.57
	.415	2.67
	.415	2.67
	37.188	2.52
	38.026	2.61
	.149	2.85
	.149	2.84
	.152	2.70
	.237	2.62
564	37.188	3.97
	.270	4.01
	38.004	4.62
565	36.415	2.85
	37.188	3.01
	.270	2.80
	.272	2.88
	38.004	3.06
	.026	2.73
	.149	2.91
	.152	3.01
	.283	2.93
	.286	2.75
566	36.179	4.25
	37.270	4.23
567	36.179	5.76
568	36.179	3.21
	.415	2.99
	37.188	3.43
	.270	3.25
	.993	3.17
	38.004	3.19
	.026	3.26
	.149	3.08
	.149	3.07
	.152	3.26
569	36.223	2.31
	.415	2.58
	37.188	2.42
	.270	2.50
	38.004	2.70
	.149	2.59

TABLE II — *Continued.*

No.	Ep.	Mag.	No.	Ep.	Mag.	No.	Ep.	Mag.
569	38.152	2.65	581	37.993	1.31	588	36.415	4.00
	.283	2.69		38.004	1.38		38.026	4.22
	.286	2.57		.015	1.46		.152	4.33
				.018	1.14			
570	36.179	4.90		.026	1.50	589	37.188	4.65
	38.026	4.78		.034	1.45		.270	4.79
				.056	1.36			
571	36.179	4.25		.149	1.21	590	36.223	2.37
	.230	4.60		.149	1.41		.415	2.63
	38.026	4.41		.152	1.31		37.188	2.52
	.152	4.42		.237	1.36		.270	2.58
							.272	2.72
572	38.283	3.20	582	38.026	4.78		38.004	2.84
	.286	3.09					.149	2.69
			583	36.179	4.25		.152	2.62
573	36.179	4.90		37.270	4.31		.283	2.56
	.240	4.13		.993	5.32		.286	2.61
	38.026	4.56		38.026	4.10			
						591	38.004	5.67
574	36.179	5.52	584	38.004	5.67			
						592	36.179	2.97
575	37.270	3.56	585	36.223	2.79		.415	3.07
	38.149	3.82		.415	2.90		37.272	3.01
	.283	3.55		37.188	2.75		38.004	2.65
	.286	3.83		.270	2.66		.026	3.03
				38.004	2.92		.149	2.96
576	37.188	5.41		.149	3.02			
				.152	2.86	593	36.179	4.90
577	36.179	3.82		.283	2.91		.319	5.04
	.415	3.78		.286	2.70			
	37.188	3.77				594	36.116	2.34
	.270	3.86	586	36.116	1.66		.179	2.16
	38.004	4.00		.179	1.73		.223	2.45
	.026	3.91		.223	1.84		.316	2.26
	.152	4.33		.415	1.81		.319	2.26
				37.188	1.94		.322	2.18
578	36.179	5.52		.270	1.90		.415	2.35
	38.026	5.58		.990	1.89		.415	2.28
				.993	1 75		37.188	2.15
579	38.149	4.72		38.015	1.68		.270	2.35
				.026	1.65		38.026	2.46
580	38.026	5.58		.034	1.64		.149	2.40
				.149	1.68		.149	2.33
581	36.114	1.18		.149	1.70		.152	2.34
	.116	1.38		.152	1.76		.237	2.23
	.179	1.22		.237	1.77			
	.223	1.31		.283	2.14	595	38.004	5.51
	.415	1.22						
	.434	1.25	587	38.004	5.51	596	36.223	2.65
	.867	1.26					.415	2.76
	37.188	1.19	588	36.179	4.25		37.188	2.52
	.990	1.47		.230	4.18		.270	2.66

TABLE II —*Continued.*

No.	Ep.	Mag.
596	37.272	3.01
	38.026	2.17
	.149	2.65
	.283	2.84
	.286	2,65
597	36.179	5.52
598	36.179	3.24
	.415	3.47
	37.272	3.13
	38.004	3.10
	.026	3.49
	.149	3.20
	.152	3.22
599	36.179	5.52
	37.270	5.49
600	36.114	1.94
	.116	1.52
	.179	1.64
	.223	1.75
	.415	1.73
	.601	1.67
	37.188	1.61
	.990	1.70
	.993	1.58
	38.004	1.62
	.015	1.61
	.018	1.54
	.026	1.58
	.034	1.58
	.056	1.67
	.149	1.57
	.152	1.57
	.237	1.57
	.283	1.72
601	37.993	5.49
602	36.179	4.90
603	37.993	4.92
604	36.179	4.25
	.415	3.96
	37.270	4.16
	38.026	4.26
605	36.179	4.90

No.	Ep.	Mag.
606	38.237	1.98
	.283	1.79
	.286	1.90
607	37.270	3.48
	38.283	3.36
	.286	3.48
608	37.270	5.65
609	38.237	2.87
	.283	2.86
	.286	2.70
610	36.179	3.92
	.230	4.23
	.415	3.82
	38.026	4.14
	.152	3.88
611	38.283	2.79
612	37.993	5.46
613	37.993	4.42
614	37.993	4.37
615	37.993	6.28
616	37.993	6.26
617	37.993	6.20
618	36.179	4.90
619	37.993	5.90
620	36.179	5.52
621	36.179	5.07
622	37.993	6.30
623	37.270	5.87
624	36.415	2.94
	38.026	3.30
	.149	3.11
	.152	3.14
	.283	3.15
	.286	3.02

No.	Ep.	Mag.
625	36.116	3.06
	.179	2.89
	.316	2.86
	.319	3.00
	.322	2.87
	.415	2.76
	37.188	3.13
	38.149	2.91
	2.83	2.89
626	36.179	4.90
627	36.179	4.90
628	36.114	1.37
	.116	1.22
	.179	1.34
	.223	1.31
	.415	1.34
	37.188	1.41
	.272	1.82
	.993	1.44
	38.018	1.35
	.056	1.52
	.149	1.47
	.149	1.50
	.237	1.47
629	38.237	2.14
	.283	2.12
	.286	2.18
630	37.993	6.24
631	37.993	4.71
632	37.993	5.92
633	36.319	4.18
	37.993	4.80
634	37.270	3.33
	.272	3.57
	38.283	3.27
	.286	3.23
635	36.116	2.56
	.179	2.51
	.316	2.51
	.319	2.47
	.322	2.47
	.415	2.45

TABLE II — *Continued.*

No.	Ep.	Mag.	No.	Ep.	Mag.	No.	Ep.	Mag.
635	36.415	2.36	648	36.319	5.46	656	38.056	1.18
	37.188	2.42					.149	1.39
	38.149	2.45	649	37.993	6.10		.149	1.32
	.152	2.39					.152	1.41
	.152	2.55	650	36.116	2.76			
				.179	2.66	657	37.993	6.15
636	36.179	4.33		.316	2.74			
	.319	4.49		.319	3.00	658	36.179	4.33
				.322	2.87		37.993	4.75
637	36.179	5.07		.415	2.56			
	37.993	5.88		.415	2.49	659	36.415	3.43
				37.188	2.63		38.149	3.36
638	36.319	5.24		.272	2.38			
	37.993	6.17		38.149	2.55	660	36.116	2.24
				.152	2.86		.179	2.28
639	37.993	6.22					.223	2.17
			651	36.179	5.52		.316	2.12
640	36.179	3.63					.319	2.03
	.316	3.41	652	37.272	2.96		.322	2.33
	.319	3.42		38.283	2.76		.415	2.27
	.322	3.37					.415	2.23
	.415	3.74	653	36.179	3.84		37.188	2.10
	37.993	3.34		.316	4.04		.272	2.15
	38.004	3.58		.319	3.99		38.056	2.12
	.149	3.52		.322	4.16		.149	2.29
				.415	4.05		.283	2.26
641	36.179	3.60		37.993	3.64			
	.316	3.30		38.149	3.73	661	37.993	6.12
	.319	3.42						
	.322	3.37	654	36.179	4.33	662	36.179	5.52
	.415	3.69		.316	4.14			
	37.993	3.49		.319	3.92	663	37.993	6.50
	38.004	3.53		.322	4.12			
	.149	3.55		37.993	4.66	664	37.272	4.28
642	36.319	4.40	655	36.179	4.25	665	37.993	5.59
				.319	4.92			
643	38.237	2.16		37.993	5.36	666	37.272	4.16
	.283	1.98						
	.286	2.00	656	36.114	1.00	667	36.114	0.80
				.116	1.04		.179	0.96
644	36.179	5.52		.179	1.10		.415	0.80
	37.993	6.08		.415	1.09		37.188	0.78
				.434	1.16		.272	0.79
645	36.179	4.33		.535	1.15		38.018	0.84
	.319	4.44		37.188	1.27		.056	0.74
				.990	1.37		.149	0.80
646	37.993	6.10		.993	1.24		.237	0.70
				38.004	1.22		.283	0.69
647	36.179	4.33		.004	1.46			
	.319	4.87		.018	1.25	668	36.179	3.71
				.034	1.38		.316	3.83

TABLE II—*Continued.*

No.	Ep.	Mag.
668	36.322	3.77
	38.149	3.58
669	36.179	4.33
	37.993	4.62
670	38.283	3.50
671	37.272	4.49
672	36.319	5.27
673	36.319	4.54
674	36.179	5.52
	37.993	5.59
675	36.319	4.58
676	36.179	5.52
	.319	5.31
677	36.179	5.52
	.319	4.96
678	36.179	4.90
	.319	4.76
679	38.283	3.05
	.286	2.86
680	36.116	2.76
	.179	2.62
	.316	2.62
	.319	2.68
	.322	2.60
	.415	2.61
	.415	2.54
	37.272	2.69
	38.149	2.62
681	36.319	4.62
	37.993	4.31
682	36.114	0.61
	.116	0.85
	.179	0.65
	.223	0 64
	.415	0.63
	.434	0.75
	.535	0.80
	.601	0.77

No.	Ep.	Mag.
682	37.188	0.63
	.990	0.65
	.993	0.61
	38.015	0.63
	.018	0.60
	.034	0.64
	.149	0.63
	.149	0.53
	.152	0.64
	.237	0.55
683	36.179	3.35
	.415	3.47
	37.270	3.40
	38.149	3.26
	.152	3 52
684	36.179	4.33
	.230	4.42
685	36.116	2.56
	.179	2.46
	.316	2.38
	.319	2.47
	.322	2.47
	.415	2.51
	.415	2.41
	37.272	2.46
	38.149	2.48
686	36.319	4.24
687	36.319	4.29
	.322	4.20
688	37.272	4.05
689	38.237	2.62
	.283	2.39
	.286	2.34
690	37.272	3.80
691	36.319	5.08
692	36.415	2.81
	37.272	3.04
	38.283	2.73
693	36.116	2.66
	.179	2.85
	.316	2.97

No.	Ep.	Mag.
693	36.319	2.84
	.322	2.74
	.415	3.12
	37.272	3.04
	38.149	2.93
694	37.272	5.45
695	36.116	2.86
	.179	3.09
	.316	3.19
	.319	3.30
	.322	3.25
	.415	3.47
	38.149	3.22
696	38.283	2.14
	.286	2.10
697	37.272	6.11
698	36.316	3.98
	.319	4.12
	.322	4.03
699	37.272	3.36
700	38.283	3.31
701	36.319	4.34
	.322	4.19
	37.272	4.34
702	36.316	3.91
	.322	3.98
	.415	3.86
	37.272	3.25
703	36.179	3.79
	.316	3.67
	.319	3.54
	.322	3.48
	.415	3.64
	38.149	3.65
704	36.319	5.43
705	37.272	4.85
706	36.179	5.52
707	36.179	3.14

TABLE II—*Continued.*

No.	Ep.	Mag.
707	36.415	3.20
	37.270	3.11
	38.149	3.09
	.152	3.05
708	36.179	4.90
709	36.319	3.84
	.322	4.08
710	36.415	2.81
	37.272	3.00
	38.283	2.66
711	38.283	3.27
712	36.316	3.49
	.319	3.42
	.322	3.37
	.415	3.47
	37.272	3.22
	38.149	3.62
713	36.179	4.90
714	36.316	3.76
	.319	3.75
	.322	3.91
715	36.316	3.58
	.319	3.64
	.322	3.58
	.415	3.52
	37.272	3.50
	38.149	3.60
716	36.319	5.00
717	36.179	4.90
	38.152	5.08
718	36.116	2.96
	.179	3.01
	.316	3.08
	.319	3.15
	.322	3.00
	.415	3.16
	37.272	3.12
	38.149	2.98
719	36.319	5.16

No.	Ep.	Mag.
720	37.272	4.10
721	37.272	3.71
722	38.283	2.52
	.286	2.26
723	35.537	4.97
	37.272	3.66
724	36.319	5.12
725	36.319	5.19
726	37.272	2.79
	38.283	2.82
727	37.272	3.42
728	37.272	5.31
729	37.272	3.39
730	36.179	2.93
	.415	3.25
	37.270	3.03
	38.149	3.12
	1.52	3.01
731	37.272	3.85
732	37.272	4.49
733	35.537	4.10
	37.272	4.21
734	37.272	3.99
735	37.272	5.45
736	35.537	2.97
	37.272	3.19
	38.149	2.95
737	36.319	3.75
	.322	3.67
	37.270	3.63
738	35.537	2.63
	36.434	2.42
	37.272	2.46
	38.149	2.52

No.	Ep.	Mag.
739	36.319	5.39
740	37.272	4.49
741	36.319	5.35
742	35.537	2.80
	37.272	2.53
	38.149	2.78
743	36.319	4.80
	37.272	4.95
744	35.537	4.19
745	35.537	4.71
746	36.179	4.90
	38.152	4.33
747	36.179	4.33
748	35.537	5.05
749	35.537	4.28
750	35.537	5.40
751	35.537	5.14
752	37.272	2.88
753	35.537	4.62
754	37.272	3.46
755	37.272	3.76
756	35.537	3.12
	37.270	2.87
	.272	3.19
	38.149	3.34
757	36.179	4.25
	.230	4.37
758	37.272	5.60
759	35.537	4.80
760	37.272	5.19

TABLE II—*Continued.*

No.	Ep.	Mag.
761	35.537	1.49
	36.114	1.56
	.179	1.44
	.434	1.42
	.535	1.41
	.601	1.29
	37.188	0.90
	.272	1.59
	38.149	1.21
762	35.537	4.54
763	35.537	4.37
	37.270	4.46
	.272	4.66
764	35.537	5.22
765	37.272	3.89
766	35.537	4.88
767	37.272	2.92
768	36.179	4.25
769	35.537	3.12
	37.270	2.95
	.272	3.19
	38.149	3.07
770	35.537	4.46
	37.270	4.53
	.272	4.76
771	36.601	2.59
	37.272	2 83
772	36.179	4.90
773	36.116	2.02
	.179	2.09
	.223	2.14
	.415	2.08
	.434	2.00
	.535	1.82
	37.270	2.20
	38.149	2.01
	.152	2.12
774	35.537	2.38
	.540	2.41

No.	Ep.	Mag.
774	38.149	2.41
775	35.537	3.81
	38.149	3.00
776	35.537	3.91
777	35.537	4.01
	38.149	3.69
778	37.272	5.05
779	37.272	3.29
780	36.601	2.52
	37.272	2.64
781	35.537	3.50
	38.149	3.39
782	37.272	3.15
783	35.537	5.32
784	36.601	3.52
785	35.540	3.29
	38.149	2.89
786	35.540	3.50
	38.149	3.41
787	35.540	3.11
	38.149	3.04
788	35.537	2.97
	38.149	3.05
789	35.537	1.85
	36.179	2.03
	.434	1.75
	.535	1.72
	.601	1.84
	38.149	1.82
790	35.537	2.16
	.540	2.08
	36.179	2.22
	.434	2.17
	.535	2.28
	.601	2.00
	37.272	2.01

No.	Ep.	Mag.
790	38.149	2.06
791	37.272	2.27
792	36.601	3.69
793	35.537	2.63
	.540	2.57
	37.272	2.74
	38.149	2.82
794	35.537	3.12
	38.149	3.17
795	35.537	3.38
	.540	3.50
	38.149	3.24
796	36.601	3.01
797	35.540	3.37
798	36.601	4.22
799	35.540	3.50
800	35.540	2.77
	36.541	2.99
	.601	2.72
801	36.601	3.32
802	35.540	2.08
	36.434	1.92
	.535	2.21
	.601	2.08
803	35.540	3.78
804	35.540	2.85
	36.601	2.96
805	36.601	5.17
806	36.541	3.98
807	36.610	3.95
808	36.434	0.91
	.601	0.93

TABLE II—*Continued.*

No.	Ep.	Mag.
809	35.540	3.43
	36.541	3.46
	.601	3.56
810	36.601	4.39
811	35.540	2.41
	36.434	2.26
	.535	2.42
	.601	2.30
	37.590	2.14
812	36.601	5.71
813	36.601	3.78
814	35.540	3.70
	.601	3.64
815	35.540	5.69
816	36.601	4.78
817	35.540	4.55
818	35.540	2.57
	30.541	2.89
	.601	2.84
819	36.601	4.50
820	35.540	3.85
	36.601	4.08
821	35.540	4.11
822	35.540	3.65
	36.601	3.60
823	36.601	3.24
824	36.601	3.40
825	35.540	4.45
826	35.540	4.11
827	35.540	4.11
828	35.540	2.99
	36.541	3.15
	.601	3.12

No.	Ep.	Mag.
829	35.540	5.08
830	35.540	3.93
831	35.540	4.67
832	35.540	4.22
	36.601	4.63
833	35.540	4.94
834	35.540	4.02
835	36.601	3.48
836	36.541	3.05
	.601	2.90
	37.590	2.63
837	35.540	5.26
838	36.434	1.50
	.535	1.30
	.601	1.19
	37.590	1.33
839	36.541	3.82
840	36.601	4.01
841	35.540	4.33
842	35.540	4.94
843	36.601	3.89
	37.590	4.02
844	35.540	4.94
845	35.540	4.80
846	35.540	4.94
847	35.540	5.46
848	36.541	3.62
849	36.601	3.40
850	37.590	4.21
851	37.590	3.53

No.	Ep.	Mag.
852	37.590	3.10
853	36.415	2.23
	.434	2.09
	.535	2.21
	.601	2.16
	.905	2.02
	37.590	2.34
	.812	1.87
854	36.535	3.20
	.541	3.36
	.905	2.97
	37.590	3.33
855	36.541	3.51
	37.590	3.64
856	36.434	1.84
857	36.541	3.78
858	36.541	4.03
	37.590	3.22
859	37.590	2.72
860	36.541	3.72
861	37.590	3.58
862	35.620	4.43
863	36.535	2.49
	37.590	2.44
864	37.590	2.96
865	35.620	2.60
	36.535	3.13
	.541	3.20
	.905	3.21
	37.590	3.16
	.812	3.43
	.874	2.72
866	37.590	5.00
867	35.620	4.56
868	37.590	2.80

TABLE II—*Continued.*

No.	Ep.	Mag.
869	37.590	4.29
870	35.620	1.74
	36.434	1.75
	.535	1.62
	.601	1.76
	.905	1.62
	37.590	1.76
	.812	1.65
871	37.590	4.79
872	37.590	5.41
873	35.620	5.02
874	35.620	3.20
	36.535	2.94
	.541	2.95
	.905	2.73
	37.590	2.89
	.812	3.18
875	37.590	4.10
876	37.590	5.34
877	37.590	3.78
878	36.541	4.03
	37.812	4.16
879	37.590	4.48
880	35.620	4.16
881	35.620	4.21
882	37.590	3.69
883	37.590	5.28
884	37.590	4.38
885	37.812	5.65
886	37.590	3.93
887	37.590	5.21
888	36.541	3.93

No.	Ep.	Mag.
889	37.590	4.33
890	35.620	2.10
	36.434	2.34
	.535	2.35
	.905	2.18
	37.590	2.01
	.730	1.81
	.812	2.17
	.874	1.88
891	37.874	5.13
892	35.620	5.12
893	35.620	5.08
	37.590	4.14
894	35.620	3.08
	36.541	3.56
	.905	3.61
	37.590	3.48
	.812	3.52
895	37.590	4.06
896	37.590	4.66
897	37.590	3.88
898	37.590	3.43
	.874	2.83
899	37.590	4.73
900	36.434	1.59
	.535	1.53
	.601	1.50
	.905	1.44
	37.590	1.48
	.812	1.42
	.874	1.48
901	35.620	4.07
902	37.590	5.14
903	36.535	2.62
904	36.535	2.54
905	35.620	4.12

No.	Ep.	Mag.
906	37.590	4.93
907	37.590	3.97
	.874	3.44
908	37.590	4.86
909	35.620	3.29
	36.905	3.92
910	36.541	3.88
	.905	3.87
	37.812	3.91
911	37.590	4.18
912	37.590	4.42
913	37.590	4.60
914	35.620	4.47
915	35.620	4.52
916	37.732	4.34
917	35.620	4.90
	37.812	4.74
918	35.620	4.60
	36.905	4.26
	37.812	4.59
919	37.732	4.62
	Unidentified stars	
920	37.874	3.87
921	38.018	6.08
922	36.179	5.76
923	37.993	6.32
924	35.540	5.91

(*To be continued.*)

A METHOD OF DETERMINING THE LUMINOSITY CURVE OF THE SOLAR SPECTRUM.

By D. W. Murphy.

A study of the relations existing between the intensities of the different colors of which the spectrum is composed may, from the nature of the investigation required, be classed as a physical problem. From the nature of the question, however, it may be said to border upon, if not to belong more strictly to, the domain of modern physiology. In a certain sense the luminosity curve may be said to represent the distribution of energy in the spectrum, the energy being expressed in terms of the effects produced upon the special sense organs of sight. This is, however, not a measure of energy according to the physical meaning of the term, and results thus obtained may, and, as we well know, do differ very widely from those obtained by measuring the energy of the spectrum in terms of the thermal effects it produces. The latter may be said to represent the total energy of radiation, and is capable of expression in mechanical units. The accuracy of such measurements depends upon the efficiency of the apparatus designed to receive and record the ether vibrations, and is independent of any peculiarities of vision which the observer may possess.

The results of observations by different observers have shown that the relations between the light-giving powers of the different colors of the spectrum are not the same when judged by different individuals. These results also show that the variations are not great except in cases of abnormal vision.

One of the difficulties, and probably the greatest one in photometric measurements of this kind, is due to the fact that there is no standard of comparison of different colored lights. The somewhat varying results which have been obtained for the visual intensities of the different parts of various spectra have

been reached by more or less modified forms of two quite distinct methods.

One of these methods is to compare the different colors with a source of light, either white or some chosen color, and to give each a value in terms of this as a standard. The other method consists in allowing the different colors of the spectrum to fall upon printed characters or other suitable objects placed upon a screen. All the light except that of the color under consideration being excluded, the intensity of the source is regulated until the characters are just visible to the observer. From the data thus obtained the relative values of the intensities for the different colors are computed.

For the above described methods and the results obtained by them we are indebted to the works of Fraunhofer,[1] Langley,[2] Abney and Festing,[3] Vierordt,[4] and others.

The first of these methods involves the difficult task of comparing photometrically two lights of marked differences in color; while the second measures the intensities of the different colored lights by the amount of each that is reflected from an object whose coefficient of absorption for different wave-lengths is not known.

The following method for determining the luminosity curve of the solar spectrum was suggested to me by the consideration of a series of measurements I had made to determine the errors in the use of the bilateral slit as a means of varying the intensity in spectrophotometric measurements.

The method consists in dividing the spectrum into a large number of separate and adjacent elements, and building up the curve by a comparison of the relative intensities of these elements. Consider the spectrum as being composed of n very narrow equal areas, and let the values of the light-giving powers of each of these areas be denoted by $i_1, i_2, i_3 \ldots . i_n$. If the

[1] FRAUNHOFER, *Gesammelte Schriften*, Münchener Akademie, 1888.

[2] *American Journal of Science*, Vol. 136. "Energy and Vision."

[3] *Phil. Trans.*, 177, 423.

[4] *Annal. Phys. Chem.*, 137, 600.

values $\frac{i_2}{i_1}, \frac{i_3}{i_2}, \frac{i_4}{i_3} \; . \; . \; . \; . \; \frac{i_n}{i_{n-1}}$ be known, the value of the intensity of any area, as the rth, is found in terms of i, and is expressed $\left(\frac{i_2}{i_1}\right)\left(\frac{i_3}{i_2}\right)\left(\frac{i_4}{i_3}\right) \; . \; . \; . \; . \left(\frac{i_r}{i_{r-1}}\right)=\frac{i_r}{i_1}$. By giving to r all values from unity to n, this expression gives the values of the ordinates of the curve in terms of some particular one chosen as a standard. The values $\frac{i_2}{i_1}, \frac{i_3}{i_2}, \frac{i_4}{i^3}$, etc., are determined photometrically by comparing the intensities of each pair of adjacent elements. By thus using only small areas of the spectrum, the difference in color of the two lights under consideration in each case is not great and does not interfere with accurate settings of the photometer.

The necessary apparatus for measurements of this kind is some form of spectrophotometer, which allows all the spectrum to be cut off except the narrow area whose intensity is to be measured. The adjustments of the apparatus must also allow an element of one spectrum to be used as a comparison light for the other spectrum, both when the two spectra are exactly superposed, and when an area of the one is superposed upon the corresponding adjacent area of the other. The requirements of the instrument are fulfilled in the Lummer-Brodhun spectrophotometer. The adjustments of the instrument and the methods of taking the observations discussed in this paper were as follows:

The instrument was so placed that the two collimators were equally inclined and pointed toward open sky. No screens of any sort were used before the collimator slit. After various preliminary experiments this was found to be the most constant light source obtainable. The observations were made during the months of June and July, when the sky was free from clouds, and the only variation in intensity was that due to the changing position of the Sun, which during the time required for any set of observations was so small as to be negligible.

The width of the collimator slit used was 0.25 mm; the angle subtended by this width of slit, when viewed through the observing

telescope, was one sixty-third part of the total angle of dispersion of the prism, between the limits within which it was possible to work. The different parts of the spectrum were brought into the field of view by turning the telescope about the axis of the instrument. Collimator *A* (see Fig. 1), which is movable by means of a tangent screw about its axis *O*, was provided with a unilateral slit so arranged that it opened in the direction from the red to the green part of the spectrum; that is, light of a lesser wave-length was brought into the field when the width of slit was increased. Collimator *B* is rigid, and was fitted with a bilateral slit.

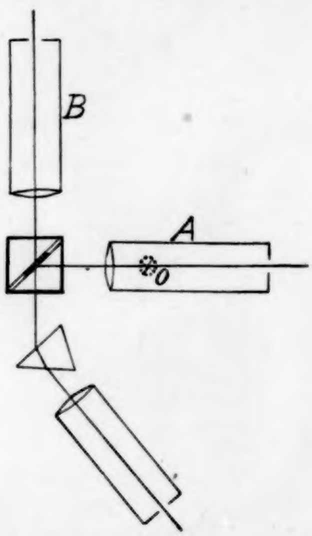


FIG. 1.

The observations were made as follows: first, the collimators were set so that the two spectra were exactly superposed, and the fields of the photometer lighted with lights of the same wave-lengths. The width of slit of *A* being 0.25 mm, slit *B* was changed until the fields showed equal intensities. Collimator *A* was then turned in the direction of the green part of the spectrum through the angular width of the slit, and the fields were again brought to the same intensities by changing the width of slit *A*. The mean values of the series of readings taken in this manner gave the relative intensities for these two elements of the spectrum. The telescope was then turned through the angle subtended by the slit width of 0.25 mm, adjustments similar to those above described were made, and a second series of readings taken. The data thus obtained gave the relative values of the intensities of the second and third areas.

In this manner the entire spectrum, from λ 6880 to λ 4500, was examined. The value of the intensity in the region λ 6880 I have called unity, and have given the values of the intensities

of the other parts of the spectrum in terms of this as a standard.

TABLE I.

Scale reading	Intensity	Scale reading	Intensity	Scale reading	Intensity
525	1.00	630	57.46	735	8.02
30	1.75	35	56.31	40	6.98
35	2.78	40	55.18	45	6.21
40	4.30	45	52.42	50	5.65
45	6.47	50	50.32	55	5.09
50	9.32	55	48.31	60	4.73
55	12.57	60	45.90	65	4.40
60	16.48	65	43.14	70	4.00
65	20.92	70	40.55	75	3.64
70	25.32	75	37.31	80	3.24
75	29.62	80	34.32	85	2.92
80	33.77	85	31.24	90	2.65
85	37.48	90	28.12	95	2.44
90	40.11	95	25.02	800	2.25
95	42.91	700	21.77	05	2.09
600	45.92	05	18.94	10	1.94
05	49.13	10	16.29	15	1.81
10	52.08	15	14.01	20	1.66
15	54.16	20	12.05	25	1.56
20	56.33	25	10.36	30	1.44
25	57.46	30	9.01	35	1.37

TABLE II.

Wave-length	Intensity	Wave-length	Intensity	Wave-length	Intensity
6880	1.00	6000	40.70	5200	31.24
6800	1.75	5900	46.62	5100	18.94
6700	3.08	5800	52.88	5000	12.05
6600	5.17	5700	56.90	4900	6.98
6500	8.75	5600	56.90	4800	4.73
6400	14.24	5500	52.42	4700	3.00
6300	20.92	5400	47.30	4600	1.94
6200	27.92	5300	40.55	4500	1.37
6100	33.77				

The results of the observations are given in Table I, the values of the intensities in the second column, the scale readings of the instrument for the regions of the spectrum having these intensities, directly opposite in the first column. These results are shown graphically by the curve (Fig. 2). This is the form of the luminosity curve of the solar spectrum for the particular prism used.

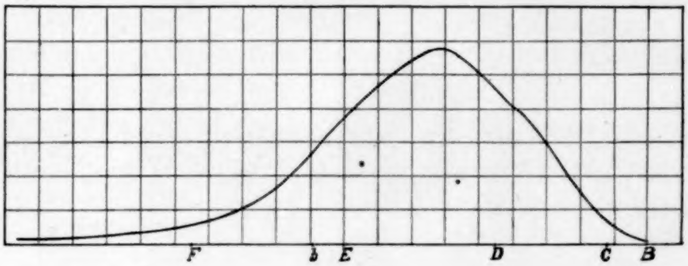


FIG. 2.

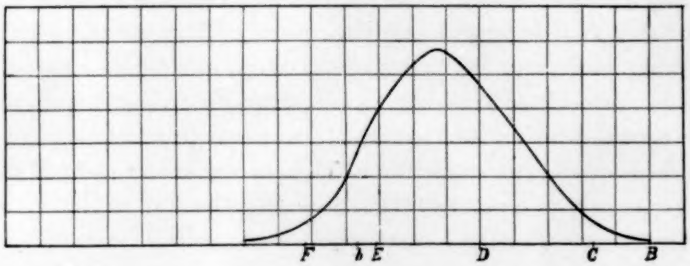


FIG. 3.

From the dispersion of the prism, together with the intensities given above, the results shown in Table II are obtained. The first column represents the wave-lengths, the second the corresponding intensities. The curve obtained from these data is shown in Fig. 3. It shows the form of the luminosity curve for the normal solar spectrum.

STANFORD UNIVERSITY,
January 1900.

SOME SPECTROGRAPHIC RESULTS OBTAINED AT THE INDIAN ECLIPSE BY THE LICK OBSERVATORY-CROCKER EXPEDITION.

By W. W. Campbell.

One of the instruments used by the Lick Observatory-Crocker Eclipse Expedition to India was an objective-grating spectrograph. It was designed for recording, continuously, on a moving plate, the changing spectrum of the Sun's edge at contacts II and III, as the edge was gradually covered and uncovered by the Moon's image; and for recording, during totality, on a fixed plate, the spectrum of the corona in the 1474 *K* region, and especially the monochromatic 1474 *K* ring.

The constants of the instrument were as follows:

A Rowland plane metallic grating, 14438 lines to the inch, ruled surface $2\frac{1}{16} \times 3\frac{3}{16}$ inches, third order, placed about five inches in front of the camera objective.

Camera objective a visual double lens of very ordinary merit, aperture $2\frac{5}{32}$ inches, focal length $20\frac{3}{4}$ inches.

Angle between the central ray incident on the grating, and the diffracted ray passing along the axis of the camera, 40°.

Center of the field at 1474 *K*, at which point, 1 mm of the spectrum equals 7.2 t. m.

Width of spectrum corresponding to the Sun's diameter, 0.20 inch.

A very light plate-holder of brass, containing a Cramer isochromatic plate $2\frac{1}{2} \times 7$ inches, was arranged to move lengthwise in a direction at right angles to the length of the spectrum, in a brass shield or slide of twice the extreme length of the plate-holder. Sensibly uniform motion, at the rate of $\frac{1}{16}$ inch per second, was imparted to the plate-holder by a small common clock-train propelled by a weight. The sensitive plate was mounted as near the front surface of the plate-holder as was

consistent with good construction. Immediately in front of the plate-holder, in contact with it, was a thin slide of brass containing a slit $\frac{1}{32}$ inch wide and $2\frac{1}{2}$ inches long, adjusted to coincidence with the axis of the spectrum. A color-screen of optical glass, stained greenish-yellow, was fixed in front of this slit, to absorb the violet of the overlapping 4th order spectrum. The slit could be removed, in an instant, by withdrawing the slide containing it; in which case no obstruction was offered to the green rays from any part of the coronal image.

The grating, brass camera tube, and plate-holder arrangements, were supported in a wooden mounting, constructed by the Observatory carpenter from my plans. The instrument was mounted on a large polar axis, in such a position that the rulings of the grating were parallel to the Sun's limb at the points of second and third contacts.

Three exposures were made as follows:

A. The plate-holder was set in motion by the clock-train, and an exposure made from 20 seconds before totality to 5 seconds after contact II. The source of light was the central $\frac{1}{32}$ inch length of the uneclipsed crescent.

B. The plate-holder was moved forward $\frac{1}{2}$ inch, and firmly clamped. The brass slide containing the $\frac{1}{32}$ inch slit was removed from in front of the plate, and a 96-second exposure on the corona spectrum was obtained, extending from 12 seconds after II to 12 seconds before III.

C. The plate-holder was unclamped and set in motion, the $\frac{1}{32}$ inch slit was replaced in front of the plate, and an exposure was made from 5 seconds before III to 16 seconds after III. The source of light was the central $\frac{1}{32}$ inch length of the reappearing crescent.

The available portion of the photograph, covering the region $\lambda\lambda$ 5150—5500, is nearly two inches in length; but as the lens is a very common one, of small field, the definition at the ends of the spectrum is necessarily poor.

The negative, which was fully developed, was found on examination to come up to expectations. The measurements

and the detailed discussion of the plate will be contained in an eclipse report, but its main features will be described below.

At the beginning of exposure *A*, 20 seconds before totality, the negative is strongly overexposed, as was anticipated, but the Fraunhofer lines are satisfactorily recorded, essentially with their usual intensities. The transition from dark lines to bright lines is well shown, and at this point many interesting features appear. Before the continuous spectrum ceased recording, many of the dark lines seem to have disappeared, or at least their intensities relative to the strength of the continuous spectrum have varied greatly. In some cases, *e. g.*, at λ 5430, the interval between a disappearing dark line and its corresponding bright line seems to be strictly continuous spectrum. In numerous other cases the dark lines and their bright counterparts coexist, apparently until the continuous spectrum becomes too weak to record itself as the necessary background for the dark lines. The relative intensities of the dark-line and bright-line spectra show marked anomalies at many points. The 1474 *K* line at the center of the plate, and the *b* group, near the edge of the plate, are the brightest lines recorded. They are of the same order of brightness, with perhaps the *b* lines the stronger, though the serious aberration effects at the edge of the plate do not permit photometric comparisons to be made. The 1474 *K* line is recorded many seconds before the bright *b* group appears at all, whereas the *b* group persists long after the 1474 *K* has disappeared. Without question, the *b* radiations proceed from a higher level in the solar atmosphere than do the 1474 *K* radiations. Similarly, the lines E_1 and E_2 at $\lambda\lambda$ 5271 and 5270 are comparable in brightness to the bright lines at $\lambda\lambda$ 5276 and 5284. The last two begin long before the E lines, but the E lines persist after $\lambda\lambda$ 5276 and 5284 have ceased to record themselves. There are many other cases equally marked. These phenomena of the dark and bright lines can scarcely fail to be of profound significance in the study of conditions existing at the Sun's edge.

About 125 bright lines are recorded in this section, of 350 tenth-meters. They are very closely related to the chromospheric

system of bright lines, as observed without an eclipse by Professor Young, at least so far as their wave-lengths are concerned. The intensities in many cases are quite different; but perhaps the chromospheric bright-line intensities vary widely at different times. It would have been exceedingly valuable, I think, to have had the chromosphere spectrum at the points of second and third contacts observed simultaneously, in the ordinary manner, at stations outside the line of totality, for comparison; and it seems desirable that such observations should be secured at some of the observatories at the time of the 1900 May eclipse. If the observations are undertaken with large telescopes and powerful spectrographs, under favorable atmospheric conditions, it may be found that the chromosphere spectrum as observed at home stations is essentially identical with the bright-line spectrum of the Sun's edge as observed at eclipse stations. Whether the results of the investigations made outside and inside the eclipse path should prove to be identical or quite different, the importance of the home observations, even if only partially successful, would in either case be very great.

The exposure *C* reproduced the above phenomena, in reverse order, at contact III.

The great strength of the calcium spectrum is shown by the fact that the H and K lines in the 4th order spectrum, after passing through the color-screen, recorded themselves faintly, and out of focus, on these photographs.

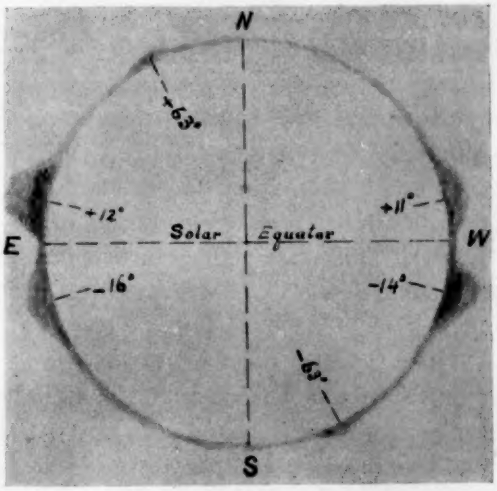
I wish to call special attention to one feature of this photograph which may be of great interest. In many, but not all, of the cases where dark and bright lines coexist, the bright lines do not have the same wave-lengths as the corresponding dark lines. The dark lines seem to be displaced toward the violet by as much as four or five tenths of a tenth-meter; and, in the case of the *b* and E groups of lines, possibly a little more. This effect is confined almost wholly to exposure *A* at contact II. Except in the case of the *b* group, it does not appear with certainty on exposure *C*, at contact III. I have endeavored, for many months,

to explain this appearance as an instrumental effect, without success. If it is purely instrumental, should not the effect be essentially identical at the two contacts? If it is due to a faulty placing of the crescents on the slit in front of the plate, so that the spectral lines were not parallel to the direction of motion of the plate, it seems to me that at II the dark lines, if shifted at all, should be shifted toward the red, and at III toward the violet. Further, another photograph obtained on a moving plate, in the same manner, with a collimating spectrograph, slit radial, shows the same effect for the lines $H\gamma$ and $H\delta$, at both II and III. This negative, obtained with a very narrow slit, is under-exposed, as was feared might be the case, and shows only the strongest of the bright lines. Its evidence supports the view that the displacement of the dark lines on the objective-grating plate represents a real phenomenon.

Assuming the reality of the displacement, we naturally consider its significance from the point of view of the effect of pressure upon wave-length. If this is a pressure effect, we conclude that the predominating absorption stratum for these lines is above the predominating radiation stratum. Whether this effect is general over the Sun's surface, or is purely local, and confined to certain lines, seems to be immaterial, so far as the interpretation of these particular observations is concerned; provided the pressure in the solar atmosphere increases toward the Sun's center. As to whether the "pressure effect," as a means of investigating the relative sources of absorption and radiation spectra at the Sun's edge, could be utilized without an eclipse, by means of large-scale apparatus, is a question.

The disappearance of the dark-line spectrum before the bright-line spectrum (at contact II) is due in part to the fact that dark lines require a continuous-spectrum background on which to manifest themselves. The continuous spectrum is weakened by the high dispersion necessarily employed, whereas the monochromatic bright-lines are not. This fact must be taken into account, both in designing the observer's instrument, and in discussing the results.

While I believe in the reality of the appearance described above, the necessity for abundant confirmation is conceded, and urged. It is only after long hesitation and delay that this brief and guarded discussion of this feature of the photograph is ventured; and further discussion of the matter, considering it either as a reality, or as an instrumental effect, will be welcomed by me.



Owing to the strong overexposure of one part of the plate, and underexposure of another part, it is unfortunately not suitable for mechanical reproduction.

The exposure *B*, with fixed plate and slit removed, for recording the coronal spectrum in the region λλ 5150–5500, was successful. The green coronal ring at λ 5303 was recorded as an elliptical ring, whose axes are 0.2 and 0.3 inch in length, respectively. This ring, first converted by projection into circular form, is reproduced in the accompanying illustration.

I have already described this ring as "extremely irregular in form" (this JOURNAL, 10, 190). In parts of it, the photographic impression is so feeble as to be scarcely traceable, whereas the inner portions of the three principal masses are fully exposed. The grouping of these masses somewhat symmetrically with reference to the Sun's equator, and near the inner edges of the Sun-spot zones, is of the highest interest. Placing this drawing of the "coronium" ring on the negatives of the inner corona secured with the 40-foot camera, there is found no evidence of any connection of these masses with the prominences, nor with the interesting curved coronal streamers which

form an enclosing hood around the principal prominences, nor with any of the main features of the inner corona. Likewise, if the four coronal extensions be carried back to the Sun's limb, the coronium masses do not seem to lie exactly at their bases. Nevertheless, it seems probable that they must be near the principal seats of coronal activity. It would be interesting to know whether these forms are shown unchanged in the photographs of the coronal rings near λ 423 and λ 399, secured by other parties.

Although the high dispersion employed was calculated to reduced the strength of the continuous-spectrum background, this was still very strong. The source of the continuous spectrum had the *same form* as the monochromatic ring at λ 5303. Each of the prominences in this ring has a strong band of continuous spectrum, of corresponding intensity, passing immediately through it.

The rulings on the grating were parallel to the line drawn from N. 26° W. to S. 26° E. The continuous spectrum along the tangents to the ring at these points was exceedingly strong, for obvious reasons, making it impossible to observe the details of the ring within some 20° of these points.

It is scarcely necessary to say that with prismatic dispersion the image of the ring would have been brighter than with the grating; and no doubt its recorded extent would have been considerably greater.

With the collimating spectrograph, slit radical, referred to above, an exposure of $1^m\ 36^s$ on the coronal spectrum was made on a fixed plate. The continuous spectrum, from $\lambda\lambda$ 3800 to 4500, was recorded out to a distance of $5\frac{1}{2}'$ on the east limb, and $4\frac{1}{2}'$ on the west limb. There is no evidence whatever of any dark lines, though the conditions for detecting them could scarcely be appreciably better. A bright line of considerable strength at λ 3987.2 and another at λ 4231 seem to be of coronal origin; but the evidence of objective-prism photographs should be of greater weight in deciding this question. The usual hydrogen and calcium lines, of great intensity, were present.

The writer is aware of the incompleteness of the foregoing discussion. Especially is it incomplete in reference to the period of transition from dark lines to bright lines, and the bearing of the evidence upon the question of a reversing layer. But in view of the extreme difficulty of the subject, further observations may with propriety precede the fuller discussion.

LICK OBSERVATORY, UNIVERSITY OF CALIFORNIA,
March 17, 1900.

Minor Contributions and Notes

SPECTROSCOPIC NOTES.

ABSOLUTE WAVE-LENGTHS, SPECTROSCOPIC DETERMINATIONS OF MOTIONS IN THE LINE OF SIGHT, AND OTHER RELATED SUBJECTS.

In the November number of the ASTROPHYSICAL JOURNAL, Professor Edwin B. Frost has a note upon "Corrections to Determinations of Absolute Wave-length," in which he states that the effect of the rotation of the Earth upon its axis, and the eccentricity of the Earth's orbit, (with the resulting motion of the observer in the line of sight), upon the wave-length of the lines of the solar spectrum, seems never to have been taken into account.

In the paper by Louis Bell on "The Absolute Wave-length of Light,"[1] he concluded that any correction due to these sources would be so small as to be negligible, especially when compared with those arising from other sources of error.

Professor Henry A. Rowland, in his paper on "The Relative Wave-length of the Lines of the Solar Spectrum,"[2] took Bell's value for D_1 as his standard of reference, and in his "New Table of Standard Wave-lengths"[3] took this value, and after applying a slight correction to it compared it with other values, weighted them all and took the mean as his final standard of reference. In a paper by myself on "The Coincidence of Solar and Metallic Lines,"[4] the differences in wave-length of a number of solar and metallic lines were given, but these were not corrected for motion in the line of sight, as sufficient data were not to be obtained from the plates from which the greater part of the measurements were made, although the effect produced was fully recognized. However, in a preliminary paper on "The Rotation Period of the Sun and other Phenomena of the Solar Atmosphere," prepared

[1] LOUIS BELL, "On the Absolute Wave-length of Light," *American Journal of Science*, March 1887.

[2] *Ibid.*

[3] *Astronomy and Astro-Physics*, **12**, 321.

[4] This JOURNAL, February 1896.

for the Harvard Astrophysical Conference in 1898,[1] the effect of motion in the line of sight of the observer and apparatus, caused by the eccentricity of the Earth's orbit and the rotation of the Earth, was carefully allowed for. The values of these factors were calculated for the latitude of Baltimore and the effect of the motion due to both causes was easily detected in my measurements. The paper was a preliminary report on an investigation which had been made in 1896 and 1897 and the reductions of which had been partially completed. For several reasons it has been necessary to postpone the completion of the work, but I am engaged upon it and expect to have it ready for publication shortly. The values due to motion were carefully calculated and diagrams made for applying the reductions graphically.

These diagrams were used in the tables of "The Arc Spectrum of Vanadium" by Professor H. A. Rowland and Caleb B. Harrison[2] and were also used in the tables of "The Arc Spectra of Titanium and Manganese," by the same authors, which are soon to be published in this JOURNAL.

Professor Frost lays rather too much stress upon the absolute wave-lengths. The values of Bell did not pretend to be accurate to such an extent as to make these corrections important, and no other determinations of absolute wave-length compare with Bell's in point of accuracy, if we except Michelson's values for some of the cadmium lines and perhaps a few lines of one or two other elements. The whole of Rowland's "Table of Solar Spectrum Wave-lengths" is based upon the value for the absolute wave-length of D_1 already determined, and it would be a very serious matter to make the change, even if the absolute wave-length of one or more lines were known with a sufficient degree of precision to justify any change.

The correction due to motion in the line of sight could be applied to Bell's measurements, but it is extremely doubtful if it would be so large as the errors arising from other sources, such as irregular changes in temperature during observations and their effect upon the instrument in ways not altogether accounted for; irregular changes in air pressure; unnoticed errors of the gratings used; small irregular errors in parts of the spectrometer; and many other things entering into the problem.

For most purposes the all important thing in a table of wave-lengths is to have the relative wave-lengths determined carefully, and

[1] Sent in but not read.

[2] This JOURNAL, 7, 273.

the effect of the Earth's rotation and orbital eccentricity upon this is very slight when compared with other sources of error.

The character of most lines in both solar and metallic spectra also places a definite limit to the accuracy of determination of both their relative and absolute wave-lengths. Careful observation of the lines in solar and metallic spectra has convinced me that both accuracy and resolving power have been pushed well toward the limit already. I am of the opinion that Professor Frost places altogether too high a value upon the determination of wave-lengths from measurements of stellar spectra. In the second spectrum of a grating of 20,000 lines to the inch and 21½ feet focus, an accuracy of a few thousandths of an Ångström unit in measurements is difficult to obtain, and an accuracy of a thousandth of a unit may be considered unattainable, except under the very best possible conditions, with the best possible instrumental equipment, and then only with solar or metallic lines of exceptional sharpness. With a dispersion not nearly so great, and definition not to be compared with this, such a degree of accuracy seems to me to be quite illusory.

It is, however, desirable that our reference points should be definitely fixed with the greatest possible accuracy. The relative wave-lengths of the better class of lines in Rowland's Table and Bell's determination of absolute wave-length are quite accurate enough for all such purposes. But for the most accurate work the lines of the solar spectrum are not stable enough for comparison lines, unless the wave-lengths in the table are considered to be the true wave-lengths of the solar lines unaffected by the Earth's motions, and corrections are made for this motion whenever the lines of the solar spectrum are used as standards of reference, and all tables of metallic lines are referred to the solar lines under these conditions.

As a matter of fact, however, the investigations I have made show that many of the solar lines are unsuitable as standards for very accurate work, and these are for the greater part the most prominent solar lines.

Most of the smaller solar lines are produced in the lower portions of the solar atmosphere, and are caused by the absorptive action upon light of matter which is on the average *ascending* over the solar surface at a rate of about 0.35 miles per second. The shaded lines, on the contrary, are produced by matter at various heights in the solar atmosphere. The shaded portions are produced by absorbing gases

at a considerable depth in the atmosphere, and the narrow central absorption line at a considerable elevation, which varies greatly for different lines. Separating these two absorption lines is a bright reversal or emission line, not very prominent, but recognizable in all metallic lines with broad shading, and conspicuous in the case of H and K and a few of the more strongly shaded lines. This emission component must be formed at an elevation somewhere between that of the matter producing the two distinctly different absorption lines. The narrow central component of the shaded lines (which is the portion of the line used in measurements), shows a *descending* motion, over the solar surface, of the absorbing matter producing it, of from about 0.18 miles per second in the case of the D lines to about a mile a second in the case of the H and K lines. The velocity in the case of the H and K lines is decidedly variable, in the cases of other shaded lines much less so.

These narrow components of the shaded lines are probably produced by meteoric matter falling into the solar atmosphere. The bright emission components may possibly be caused by the downrush of this meteoric matter through the denser portions of the chromosphere such that where it meets the uprush of the matter already referred to, the impact of the collisions or the friction caused produces an intense emission of certain lines in this region of the chromosphere. This may be the true seat of the so-called "reversing layer."

Another factor which affects the wave-lengths of solar lines, but to a less extent in general, is pressure. In the case of the central lines of H and K, the pressure is probably not far from zero, or at least small; in the case of the D lines it is about 1½ atmospheres, and in the case of most of the small lines of iron and the other elements it is two or three atmospheres. The shaded component of the shaded lines is probably produced by matter at a pressure as great as or greater than this. The change in wave-length of most solar lines from this cause is slight, but in a few cases, as in the small sodium lines, it is considerable.

Such facts as these rather impair the usefulness of many solar lines for standards of reference where measurements of velocity in the line of sight are in question, and although the smaller solar lines and most of the others probably have a very constant actual wave-length, still this wave-length is not that which these same lines would have if

the matter producing them were at rest and at atmospheric pressure, and if the point on the Earth's surface where the observer may be were free from motion.

Stellar spectra, however, are not compared with solar, but with metallic or gaseous spectra, and the wave-lengths of lines from these sources as usually obtained may be considered to be quite constant, or at least such within reasonable limits. It is desirable, nevertheless, that the wave-lengths of these lines, used as standards, should be determined accurately, and that this determination be made with the observer at rest with respect to the Sun (in a radial direction) by comparison with the lines of the solar spectrum, and that from these lines a careful selection be made.

In my investigation the values thus obtained were determined for a carefully selected list of iron and other lines, and will be published when the work is completed.

The same corrections were in greater part applied to the wave-lengths of the lines in the spectrum of titanium and manganese determined by Rowland and Harrison soon to be published.

What I have said regarding the effect of motion in the line of sight and pressure in the solar atmosphere, upon the wave-length of lines in the solar spectrum, also holds true of stellar spectra; and in some cases the effect may be considerably greater than in the case of the solar spectrum, as in those stellar atmospheres where the disturbances are greater; or the vertical circulation more intense; or the density or extent of the atmosphere greater than with the Sun. As this matter is to be taken up at considerable length in the near future it is not necessary to discuss it to any extent now.

It is, however, well to say that the effect of rotation of the star upon its axis will be merely to broaden the lines in its spectrum, without materially affecting their position, while radial or nearly radial disturbances or atmospheric circulation may considerably affect the wave-length of lines in its spectrum. The effect of a very active vertical circulation will probably be, as seems to be the case with the Sun, to shorten the wave-length of the smaller metallic lines, and of those lines which are produced by absorption in the lower portions of the star's atmosphere, in view of the fact that the gases producing this absorption take part in the uprushes concerned in this vertical circulation. The study of the appearance of the lines in the solar spectrum leads to the conclusion that the uprush is almost wholly

responsible for the absorption producing these lines. The return current or fall of the material concerned in these uprushes seems to be but slightly effective in the production of absorption lines, the material of these return currents being probably too cool to be of influence. This also best fits in with the most approved solar theories.

The more permanent gases which may constitute the bulk of the true solar atmosphere may not indicate by their spectral lines motion in any predominant direction, though I am not in a position at present to state that this is actually the case. However, the lines most prominent in the solar chromosphere are likely to show a change of wavelength which indicates a down-rush of matter, not under very great pressure, but rather attenuated. In the case of the more prominent lines this velocity may be quite variable. This is upon the supposition that the definition of the lines in stellar spectra is such as to admit of the distinguishing of the narrow central components from the shading of these lines. Such is not likely to prove the case, and hence only the uprush will be measured. This downward or falling velocity is probably not directly connected with the vertical atmospheric circulation referred to, but is more likely to be of meteoric origin. Hence the metallic lines affected will for the greater part be those whose intensity is much greater in the spark than in the arc spectrum, the rush of meteoric matter through a star's atmosphere probably giving a spectrum more nearly like that of the spark than the arc, seeing that it is the result of friction or collisions.

These conclusions are certainly indicated by the appearance and wave-lengths of these lines in the solar spectrum and their prominence in the chromosphere. Hence the facts regarding the behavior and appearance of certain lines in the solar spectrum should make us wary about accepting the presence or predominance of arc or spark lines in stellar spectra as an indication of the temperature of the star's atmosphere, since both the extent and density of a star's atmosphere, and the violence or quietness of its atmospheric circulation, and the amount of meteoric matter falling into its atmosphere, may influence the star's spectrum much more than its temperature alone.

The factors mentioned may also to some extent affect our conclusions regarding the real motion of a star in the line of sight, although if the determinations are based upon measurements of the lines of hydrogen or other permanent gases they are probably less likely to be thus affected. This, however, is a matter to be investigated.

The effect of pressure upon the wave-length of lines as a disturbing influence in measurements of motion in the line of sight will probably be slight. As I have stated, the solar lines indicate pressures of from little more than zero to only two or three atmospheres, though the shading of the stronger lines may be produced at a greater pressure. While some of the stars may have atmospheres of much greater extent and under greater pressure than that of our own Sun, it is doubtful if the seat of absorption of measurable lines will be at a pressure much, if any, greater, and as but comparatively few metallic lines have a large pressure shift per atmosphere, the effect of pressure will be of little importance in most cases.

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PHOTOGRAPHIC NOTES.

IN astronomical and spectroscopic photography, halation may sometimes prove troublesome. This effect may be gotten rid of by using doubly and triply-coated plates, but in some respects they are troublesome, especially in the matter of thorough fixing, and furthermore are not always at hand when wanted. Many receipts have been given for backing plates to prevent halation. These generally include essential oils for rendering the index of refraction of the medium which contains the dark pigment used, the same as glass. Both the backing of the plates and its removal later is liable to prove troublesome.

I have found water-color lampblack, in moist pans, or better still, in tubes, to answer perfectly as a preventive of halation. It may be that the gum arabic and honey with which the pigment is mixed render the refractive index of the medium nearly the same as glass; at any rate, it has proven effective. To apply and afterwards remove the black is very easy. It is only necessary to squeeze out some of the color, and with a moistened brush render it of about the consistency of paste, and then apply it to the back of the plate. It drys very rapidly and can be readily removed before developing by washing under the top, or by other means. The film side should first be thoroughly wet, so that there may be no danger of the paint getting upon the surface of the film.

Some years ago an extensive series of experiments led me to adopt a developing formula, which has proven to be so satisfactory for all

around work that I have had no occasion to change it. As it may prove useful to other workers in astrophysics, I have thought it might be well to give the formulae I have been using. Some of the essentials of a good developing formula are that it should be convenient, work cleanly, and with nearly absolute uniformity, both when fresh and old; and also that it should readily lend itself to any changes desired in the treatment of different subjects without the expenditure of too much trouble. The *normal* developer which I employ can be used over many times, and, instead of becoming harsh in its action, will give the same gradations, or contrasts in the negatives when old as when fresh, although when old its action is, of course, slower. It can be mixed without a graduate if necessary, and may even be left around in open tumblers for weeks. This, however, is not advisable, and, like all developers, it should be kept from very strong light, and the necks of bottles in which it is contained should be kept clean and no developer allowed to dry upon them.

A small amount of alcohol is used for two reasons: first, the film is more thoroughly wetted at the start, thus rendering "flowing marks" less liable to occur; and second, the developing action begins sooner and proceeds somewhat more rapidly. Potassium ferrocyanide is also used, because it was found in practice that when included the developer kept better and worked with greater clearness and delicacy. It may, however, be left out without serious consequences. The developer is a concentrated one, and, to prevent spoiling the fixing bath, the plate should be washed thoroughly before fixing, or, much better still, some sodium sulphite used with the fixing bath.

In practice I have found it convenient, where it is desired to secure the best possible results, and the exposure is uncertain, to have conveniently at hand a *normal* developer and two correctors for under and overexposures. The developer for underexposures will give a thin negative with much detail, and will become more harsh in its action or give greater contrast after it has been used a few times. The developer for overexposures will give a negative with very great contrast, but this contrast will somewhat decrease with further use of the developer. As already stated, the *normal* developer will remain constant in its action.

When changing from a developer with more potassium bromide to one with less it is important to rinse the plate well before changing. When the change is of the opposite character it is hardly necessary.

Alum should be kept out of the fixing bath, and only used after one has fixed and rinsed the plate well. It is sometimes useful in very hot weather.

FORMULAE.

(H)	Hydrochinone	1 part
	Sodium sulphite (crystals)	5 parts
	Water (distilled if obtainable)	25 parts
	Alcohol	¼ ± part
(C)	Potassium carbonate	1 part
	Potassium ferrocyanide	1 part
	Water	12 parts
(b)	Potassium bromide	1 part
	Water	10 parts

NORMAL DEVELOPER.

(N) 25 cc (C) + 75 cc (H) + 10 drops (b) (from pipette).
= 1 oz. (C) + 3 oz. (H) + 10 drops (b) (from pipette).

The amount of bromide for normal developer should be according to class of work. For most spectroscopic work 15 drops will be found best for above amount of developer, but 10 drops is about right for most kinds of photographic work.

UNDEREXPOSED WORK (thin negatives).

(U) 25 cc (C) + 75 cc (H), no bromide.
= 1 oz. (C) + 3 oz. (H), no bromide.

OVEREXPOSED WORK (great contrast).

(O) 20 cc (C) + 10 cc (b) + 70 cc (H).
= 1 oz. (C) + ½ oz. (b) + 3½ oz. (H).

In very hot weather the developer may be diluted to a considerable extent if desired.

If for any reason it is desired to develop more rapidly the amount of accelerator (C) may be increased, the amount of the bromide solution being increased in the same proportion. The contrasts will be about the same as with the normal developer already given, but the grain of the plate may be coarser and some tendency to fogging of the plate may result.

The fixing bath used is about as follows, the exact proportions not being a matter of much consequence providing it is made strong enough.

FIXING BATH.

Sodium sulphite	1 part
Sodium hyposulphite	5 parts
Water	25 parts

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THE USE OF THE LINES OF TITANIUM FOR COMPARISON SPECTRA AND THEIR PROMINENCE IN THE CHROMOSPHERE.

PROFESSOR FROST'S note upon the use of titanium spark lines for comparison spectra is of much interest. This element, though one of our most refractory substances, seems to give a very marked spark spectrum, and is, moreover, in many ways one of the most prominent solar elements. The smaller lines are remarkably prominent in Sun-spot spectra, and the stronger lines, especially those prominent in both spark and arc spectra, form a very large proportion of the strong lines in the chromosphere and extend to a great elevation.

I first discovered the importance of titanium in chromosphere and prominence spectra from some of Professor Hale's photographs of prominence spectra taken several years ago. These showed the titanium lines at λ 3685.339, 3759.447, and 3761.464 to be among the most conspicuous of the chromosphere and prominence lines, and to extend almost if not quite as far out as the lines of hydrogen and helium, although not so far as those of calcium (H and K). In Evershed's eclipse spectra, taken in India in 1898, the ultra-violet part of the spectrum is crowded with the strong titanium lines which are prominent in spark spectra, and of these many are among the strongest of all the chromospheric lines. We find, in fact, that titanium furnishes more strong ultra-violet lines than any other element, there being (together with many weaker lines) about thirty of importance between λ 3340 and λ 4000.

There are, furthermore, several extremely strong manganese and chromium lines prominent in the spark, while the rare element scandium furnishes a remarkably large number of lines, some of which are

also fairly conspicuous. The characteristic spark lines of iron do not play so striking a part in the chromospheric spectrum, and of those which do appear the strongly shaded lines are the most important. The same is true of nickel, cobalt, and magnesium.

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OPPOSITION OF *EROS* IN 1900.[1]

THE opposition of *Eros* during next autumn will afford opportunities for observations of especial interest. The near approach of the planet to the Earth will permit the solar parallax to be determined, while the great variations in phase and distance will give unusual value to photometric observations obtained at this time. The ephemeris of Dr. Millosevich, published in the Berlin *Jahrbuch* for 1902, provides the means for discussing the measures of position of *Eros*. The annexed table, which is based on this ephemeris, furnishes a part of the material required for the investigations mentioned above. The date is given in in the first column. The right ascension and declination for 1855, for Berlin midnight, are given in the second and third columns. This epoch is selected for convenience in identifying *Eros* by comparison with the stars in the *Durchmusterung*. The daily motions in right ascension, expressed in seconds of time, and in minutes of arc when reduced to the equator, are given in the fourth and fifth columns. The daily motion in declination, and the total motion expressed in minutes of arc, are given in the sixth and seventh columns. These quantities are important in planning observations for parallax, especially those made photographically. The logarithm of the distance from the Sun has been kindly furnished by Dr. Millosevich, and is given in the eighth column. The logarithm of the distance from the Earth is given in the ninth column. From this it appears that its distance when nearest the Earth is less than a third of that of the Sun from the Earth. This minimum occurs on December 26, nearly two months after opposition, which takes place on October 30. The phase angle between the Sun and Earth, as seen from *Eros*, is given in the tenth column. There are few asteroids for which this angle much exceeds 30°. In the table, beginning with the value $37\overset{\circ}{.}5$ it gradually diminishes to $28\overset{\circ}{.}3$ at about the time of opposition, and then gradually

[1] *Harvard College Observatory Circular* No. 49.

EPHEMERIS OF *EROS.*

Date 1900–1901	R. A. 1855	Dec. 1855	Daily Motion R. A.	Daily Motion R. A.	Daily Motion Dec.	Daily Motion Tot.	log r	log Δ	Phase	Mag.	C. Mag.	M. Tr.	Aberr.
	h m	° ′	s	′	′	′			°			h. m.	s.
Sept. 1	2 18.9	+33 27	+ 70	+15	+22	25	0.1830	9.9088	37.5	11.85	12.33	15 39	404
9	27.5	+36 25	+ 60	+12	+22	25	.1761	.8692	36.6	11.62	12.08	15 16	369
17	34.6	+39 26	+ 45	+ 9	+23	25	.1688	.8285	35.4	11.38	11.80	14 52	336
25	39.2	+42 29	+ 25	+ 5	+23	24	.1613	.7871	34.0	11.13	11.51	14 25	305
Oct. 3	40.8	+45 30	− 1	0	+22	22	.1535	.7456	32.5	10.89	11.23	13 55	278
11	38.7	+48 20	− 32	− 5	+20	21	.1455	.7048	31.0	10.64	10.93	13 21	253
19	32.0	+50 52	− 68	−11	+17	20	.1374	.6654	29.6	10.40	10.65	12 43	231
27	20.6	+52 49	−101	−15	+12	19	.1291	.6286	28.6	10.18	10.40	12 0	212
Nov. 4	5.7	+53 55	−121	−18	+ 5	19	.1207	.5954	28.3	9.97	10.18	11 14	196
12	1 49.3	+54 0	−120	−18	− 3	18	.1124	.5666	29.1	9.79	10.02	10 26	184
20	35.0	+53 0	− 92	−14	−11	18	.1041	.5430	30.9	9.63	9.92	9 40	174
28	25.8	+51 2	− 45	− 7	−18	19	.0961	.5248	33.6	9.49	9.86	8 59	167
Dec. 6	23.5	+48 22	+ 9	+ 2	−22	22	.0884	.5117	36.8	9.39	9.85	8 25	162
14	28.2	+45 16	+ 60	+11	−24	26	.0812	.5033	40.2	9.31	9.88	7 58	159
22	39.3	+41 56	+106	+20	−26	33	.0746	.4994	43.7	9.26	9.93	7 38	157
30	56.0	+38 31	+143	+28	−26	38	.0686	.4994	47.0	9.23	10.00	7 23	158
Jan. 7	2 17.1	+35 7	+172	+35	−26	44	.0636	.5029	49.8	9.22	10.07	7 13	159
15	41.6	+31 43	+194	+41	−25	48	.0595	.5101	52.2	9.24	10.17	7 6	161
23	3 8.6	+28 23	+210	+46	−25	52	.0566	.5210	54.3	9.28	10.27	7 1	165
31	37.3	+25 7	+219	+50	−24	55	.0548	.5353	56.1	9.34	10.38	6 58	171

increases, until on January 31, it attains the extraordinary value of $56^{\circ}.1$, becoming even greater later. The photometric magnitude, neglecting the phase and assuming that the light is inversely proportional to the squares of the distances of the Earth and the Sun, is given in the eleventh column. It is based on the measures described in *H. C. O. Circular* No. 34, from which it appears that the magnitude would be 11.39 at a distance of unity from the Sun and Earth, and that the photographic magnitude is 0.6 fainter than the photometric. It will be noticed that these last values are nearly 0.8 fainter than those given by Dr. Millosevich, who based his magnitudes on visual observations. As the magnitude 9.5 in the *Durchmusterung* is about 10.5 on the photometric scale, this difference is readily explained. The difference becomes still greater if we apply a correction for phase. This correction, in the case of the asteroids, is about $0.03\,p$, in which p is the phase angle. If we assume that this law can be applied to *Eros* for angles as great as 56° we obtain the corrected magnitudes given in the twelfth column. The phase angle in the observations described in *Circular* No. 34 is $21^{\circ}.2$. The magnitude at distance unity therefore becomes $11.39 - 0.64 = 10.75$. The approximate mean time of meridian transit is given in the thirteenth column, and the aberration time in the fourteenth.

As an example of the use of this table, let us consider the most favorable conditions for determining the solar parallax. It soon appears that this problem is by no means a simple one. If we select the end of December, when *Eros* is nearest the Earth, we find that meridian transit occurs so early in the evening that *Eros* cannot be photographed far east of the meridian. Moreover, the motion both in right ascension and declination is so great that if the telescope is made to follow the stars, *Eros* will trail so rapidly over the plate that it may not leave any impression on it. If the total diurnal motion is $24'$, the motion will be $1''$ a minute. If, then, the diameter of the image is $2''$, *Eros* cannot be photographed unless an exposure of two minutes is sufficient. In such a case it may be necessary to make the telescope follow on *Eros* and not on a star. All the stars will then appear as short trails which are easily bisected. If the motion of *Eros* is large, its position with relation to the comparison stars will differ greatly when east and when west of the meridian. Moreover, it will be necessary to measure the total motion, and after subtracting the large motion of *Eros*, determine the small remaining parallax. During

the latter part of January *Eros* culminates at nearly the same time on successive nights, and will thus be favorably situated for observations west of the meridian for several weeks. The path of *Eros* has a loop extending over about 13° in right ascension and 20° in declination, and with a center at about R. A. $2^h\ 5^m$, Dec. $+51°$. The point of crossing is at R. A. $2^h\ 21^m.9$, Dec. $+34°.25'$ (1855). It is therefore not far from the stars $+34°.447$, mag. 9.3, and $+34°.448$, mag. 8.0. *Eros* will pass through the point of crossing on September 3, 1900, and again on January 8, 1901. Photometric observations, if made on these dates, will have especial value, since the same comparison stars can be used for both.

A photograph of *Eros* was obtained on September 6, 1898, with the 11-inch Draper telescope, whose focal length is 153 inches. Stars of the ninth magnitude are readily photographed with this instrument in 5 seconds. The exposure was 10 minutes, the daily motion 18′, and the computed magnitude 12.1. Allowing for the difference in motion it would be equally difficult to photograph *Eros* on this date and on September 17, 1900.

EDWARD C. PICKERING.

February 14, 1900.

A REMARK ON THE ARTICLES ON THE DENSITY OF THE ALGOL STARS IN THE ASTROPHYSICAL JOURNAL, VOL. 10, NO. 5.

IT has long been known that the mean density of the two components can be deduced from the observations of the light-variation of the Algol stars. This question was discussed thoroughly, and in principle completely, by Mériau in *Comptes rendus*, **122**, 1254. In my article on "Doppelsterne" in Valentiner's *Handwörterbuch der Astronomie* (Bd. I, 694–695), mention is also made of this idea, which naturally suggests itself.

If we retain the symbols employed by Roberts, and further let δ_1 and δ_2 represent the densities of components 1 and 2, δ the *mean* density of the two components, and D that of the Sun, whose radius in astronomical units is R, we shall have

$$\frac{\delta_1}{D}=\frac{R^3}{p^3t^2}\cdot\frac{m_1}{m_1+m}\ ;\quad \frac{\delta}{D}=\frac{R^3}{q^3t^2}\cdot\frac{m_2}{m_1+m_2}\ ;\quad \frac{\delta}{D}=\frac{R^2}{(p^3+q^3)t^2}\ .$$

The last formula alone, which was also derived in my article referred to above, can be computed without any hypothesis. The first two

formulae only were employed by Roberts, who, however, used an incorrect factor. It would appear that the Sun's diameter was taken instead of its radius, for his factor 0.0092 must be replaced by 0.00465 (D being placed equal to unity).

In conclusion, it is by no means intended that these lines shall detract from the interest which is certainly merited by the numerical results of the articles referred to.

H. SEELIGER.

MUNICH, February 1900.

THE SYSTEM OF CAPELLA.

ONE of the most interesting among recent astronomical discoveries is announced in Professor Campbell's note on "The Spectroscopic Binary *Capella*," published in the October 1899 number of this JOURNAL.[1]

I have plotted the data given in Professor Campbell's note, and find that a period of about 105 days will satisfy the six observations recorded. A shorter period, though not absolutely precluded, is clearly improbable. A longer period ($200^{d} \pm$) cannot be brought into harmony with Vogel's spectrographic observations of 1888–9.[2] The indicated range in radial velocity is about 57 km per second, and the orbit is not far from circular. I have made no attempt to deduce the elements by "least squares," as further observations will doubtless be soon available.

Professor Vogel found no evidence of change in the radial motion of *Capella*. His mean result for this motion (+15.5 English miles per second) should therefore hold good for the center of mass of the system; and we may further assume that the two components are nearly equal in mass, as well as in brightness.[3]

Adopting Elkin's value for the star's parallax, viz., 0".081, I infer that the components will at times be separated by more than 0".04.

[1] The binary nature of this star was independently detected by Mr. H. F. Newall (*Observatory*, December 1899, p. 436).

[2] Reproduced in the *Observatory*, No. 181, p. 374. Vogel's result for the radial velocity of *Capella* considerably exceeds the deduced value for the motion of the center of mass, based on the 200-day period.

[3] The non-detection of changes in the star's spectrum by Professor Vogel is not easily explained; and Professor Campbell's remarks on the dissimilar spectra of the components tend to heighten the difficulty. Evidently from any point of view, the case of *Capella* is of great interest.

Hence Michelson's interference apparatus,[1] fitted to one of our larger telescopes, should furnish reliable measures of the Capella system, thus affording material for a complete knowledge of the orbit and absolute masses, besides yielding a new value for the star's parallax. Since *Capella* is now favorably situated, while suitable apparatus is easily improvised, I trust that no time will be lost in the making of tentative experiments. With the assigned period the components would attain their maximum angular distance about January 1, and again near the end of February 1900.

I have hitherto assumed that the inclination $(90^\circ - i)$ of the orbit plane to the line of sight is not large. This is a natural supposition; but the following deductions lead to a different result, and one which, from our present standpoint, is of great interest.

If m denote the mass of a binary star in units of the Sun's mass, P being the period of revolution in *days*, and $V_0 = V$ cosec i the *relative* mean orbital velocity in kilometers per second, we shall have:

$$m = [\bar{7}.01645]\, PV^3 \operatorname{cosec}^3 i,$$

where the semi-axis major of the Earth's orbit is taken as 149,480,000 kilometers.[2] Assuming equal masses for the components, and putting $V = 56$, $P = 105$, $i = 90^\circ$, we find $m = 1.9$ for the system of *Capella*.

Now this value of the mass is notably less than that which we should be led to assign from a consideration of the star's actual brightness. *Capella* is intrinsically about five times brighter than *Sirius*, twenty-one times brighter than *Procyon*, and eighty-seven times as bright as *α Centauri*.[3] And the masses of *Sirius*, *Procyon*, and *α Centauri* are respectively equal to 2.4, 3, and 2 Sun-masses, according to the reliable data supplied by Auwers, Gill, Elkin, and See. Taking

[1] This is, in brief, a cap fitting over the telescope objective and provided with two parallel slits whose breadth and distance apart can be varied at will. A simple cap with fixed slits and a graduated band for the measurement of position-angles will serve the purpose.

[2] In the deduction of approximate or limiting values for the masses of spectroscopic binaries, this formula will be found very convenient. In general, the *most probable* value (m_p) of the mass will be obtained by putting $i = 60^\circ$. The corresponding formula is:

$$m_p = [\bar{7}.20386]\, PV^3.$$

[3] These comparisons are based on the Harvard photometric magnitudes of the four stars. The adopted parallaxes are those of Gill for *Sirius* and *α Centauri*, and of Elkin for *Procyon* and *Capella*.

these and other related facts into consideration, we may assume with much confidence that $m > 6$, in which case $a > 0''.064$, and $i < 43°$, for the system of *Capella*.

If the foregoing deductions are sound — and they certainly possess a high measure of probability — it will be possible to study this interesting system by interference methods at almost any epoch, with the large refractors of the Yerkes and Lick Observatories.[1]

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January 4, 1900.

Addendum.— Since the foregoing paper was written, Professor Campbell has announced his preliminary results relative to the orbit of the Capella system.[2] These results are in remarkably close agreement with those deduced by the writer. Using Professor Campbell's data ($P = 104^d.1$, $V = 57$ km per second), I find:

$$a = 0''.044 \operatorname{cosec} i,$$
$$m = 2.00 \operatorname{cosec}^3 i,$$

the orbit being supposed circular. Putting $m = 6$, as already assumed, the maximum and minimum distances of components will exceed $0''.064$ and $0''.046$, respectively. The actual value of a will probably be found to lie between $0''.1$ and $0''.25$. Hence this interesting system should come within the range of direct observation in our larger telescopes — a suitable absorbent screen being used to reduce the apparent dimensions of the diffraction-disks.

J. M. B.

[1] It may be worth while to directly examine *Capella* with high powers, under exceptionally favorable conditions.

[2] *Observatory*, February 1900, p. 92. See also Mr. Newall's interesting letter in the same number.